1.1 Complex Numbers: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The complex number z satisfies $|z - (3 - 2i)| = 4$.

(a) Describe the locus of z geometrically, and write its equation in Cartesian form. **(4)**

(b) Find the greatest value of $|z|$ and the least value of $|z|$. **(3)**

(Total for Question 1 is 7 marks)

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2 The complex number z is $z = 1 - \sqrt{3}i$.

(a) Write z in modulus-argument form $r(\cos\theta + i\sin\theta)$, with θ in degrees. **(3)**

(b) Use de Moivre's theorem to find z^9, giving your answer in the form $a + bi$, with a and b exact. **(4)**

(Total for Question 2 is 7 marks)

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3 The cubic equation $z^3 + pz^2 + qz - 17 = 0$, where p and q are real constants, has a root $-4 + i$.

(a) Write down another complex root of the equation, with a reason. **(2)**

(b) Find the third root of the equation. **(3)**

(c) Find the values of p and q. **(4)**

(Total for Question 3 is 9 marks)

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4 This question is about De Moivre's theorem.

(a) State De Moivre's theorem for a positive integer n. **(1)**

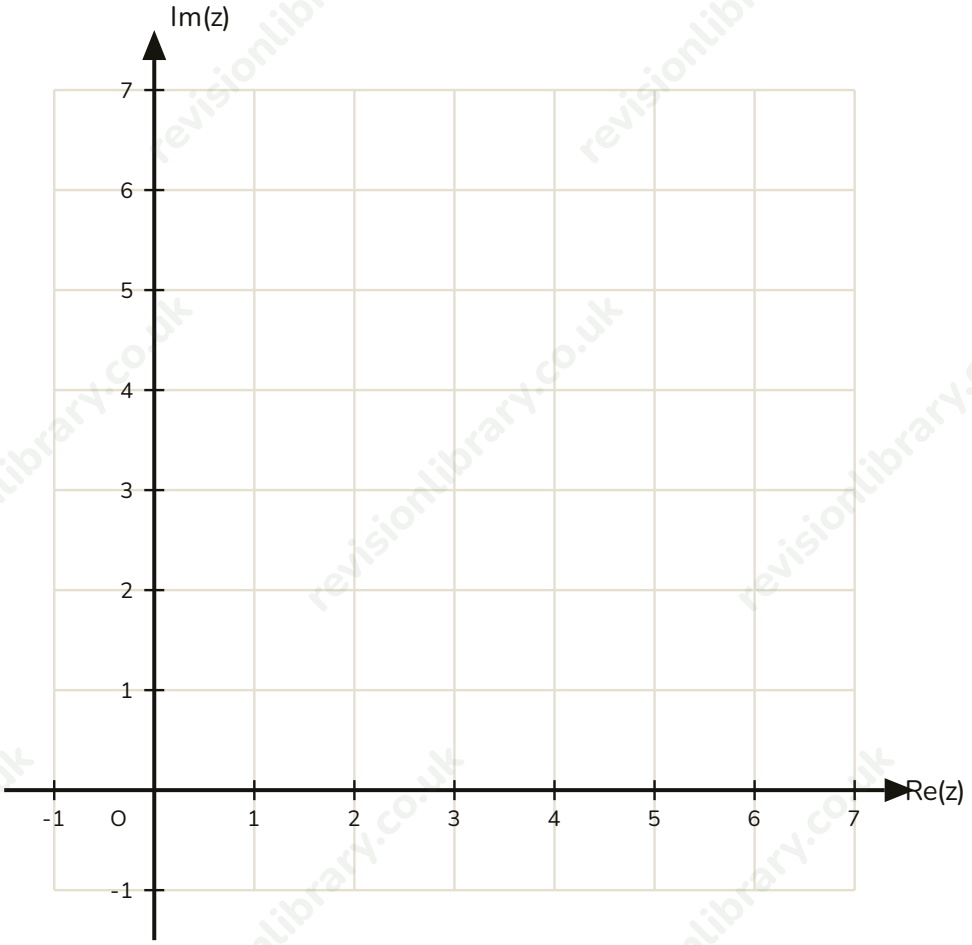
(b) Hence find, in the form a + bi, the value of $(\cos(\pi/6) + i \sin(\pi/6))^9$. **(3)**

(c) Find the smallest positive integer n for which $(\cos(\pi/6) + i \sin(\pi/6))^n$ is a positive real number. **(3)**

(Total for Question 4 is 7 marks)

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5 The region R in the Argand diagram is defined by $|z - 3| \leq 3$ and $0 \leq \arg(z) \leq \pi/4$.



(a) Sketch the region R on an Argand diagram. **(3)**

(b) Show that the circle $|z - 3| = 3$ has polar equation $r = 6\cos(\theta)$, for $-\pi/2 \leq \theta \leq \pi/2$. **(3)**

(c) Hence find the exact area of the region R. **(6)**

(Total for Question 5 is 12 marks)

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6 This question uses De Moivre's theorem to derive a multiple-angle identity.

(a) Use De Moivre's theorem and the binomial expansion of $(\cos\theta + i\sin\theta)^5$ to show that $\cos(5\theta) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$. **(5)**

(b) Hence solve, for $0 < \theta < \pi$, the equation $16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta) = 0$, giving each answer as an exact multiple of π. **(4)**

(Total for Question 6 is 9 marks)

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Mark scheme · 1.1 Complex Numbers: 1-hour Pack

Question 1

(a) a circle with centre (3, -2) **B1**

(a) radius 4 **B1**

(a) writes $z = x + iy$ and uses $|z - w|^2$ **M1**

(a) $(x - 3)^2 + (y + 2)^2 = 16$ **A1**

(a) Answer: $(x - 3)^2 + (y + 2)^2 = 16$

(b) distance of the centre from the origin = $\sqrt{13}$ **M1**

(b) greatest = $\sqrt{13} + 4$ **A1**

(b) least = $|\sqrt{13} - 4|$ (the origin lies inside the circle when $\sqrt{13} < 4$) **A1**

(b) Answer: greatest $\sqrt{13} + 4 = 7.606$, least 0.3944

Question 2

(a) finds the modulus **M1**

(a) r = 2 **A1**

(a) θ = -60 degrees (accept 300 degrees) **A1**

(a) Answer: z = 2(cos -60 + i sin -60) degrees (or 2(cos 300 + i sin 300))

(b) $z^9 = (2)^9$ (cos(9 x -60) + i sin(9 x -60)) **M1**

(b) modulus 512 and argument -540 degrees (or an equivalent argument, e.g. 2700 or 180 degrees) **A1**

(b) uses exact values of cos(180) and sin(180) **M1**

(b) -512 **A1**

(b) Answer: -512

Question 3

(a) -4 - i **B1**

(a) complex roots of a polynomial with real coefficients occur in conjugate pairs **B1**

(a) Answer: -4 - i

(b) uses the product of the roots: (α)(α*)(r) = -(-17) **M1**

(b) $((-4)^2 + 1^2)$ r = 17 **A1**

(b) r = 1 **A1**

(b) Answer: 1

(c) uses sum of roots = -p **M1**

(c) p = 7 **A1**

(c) uses the sum of products of pairs = q **M1**

(c) q = 9 **A1**

(c) Answer: p = 7, q = 9

Question 4

(a) $(\cos \theta + i \sin \theta)^n = \cos(n \theta) + i \sin(n \theta)$ **B1**

(a) Answer: $(\cos \theta + i \sin \theta)^n = \cos(n \theta) + i \sin(n \theta)$

(b) uses De Moivre with n θ = 9 x π/6 = 3pi/2 **M1**

(b) cos(3pi/2) = 0 **A1**

(b) sin(3pi/2) = -1, giving answer -i, cao **A1**

(b) Answer: -i

(c) requires n x π/6 = 2k π for a positive integer k **M1**

(c) n = 12k **A1**

(c) smallest positive integer n = 12, cao **A1**

(c) Answer: n = 12

Question 5

(a) circle centre 3, radius 3, passing through the origin, drawn correctly **B1**

(a) half-lines arg(z) = 0 and arg(z) = π/4 drawn from the origin **B1**

(a) region R (bounded by the circle and lying between the two half-lines) correctly identified/shaded **B1**

(a) Answer: Region bounded by the circle |z-3|=3 and the half-lines arg(z)=0, arg(z)=π/4, with the origin on the boundary

(b) writes $|z-3|=3$ as $(x-3)^2+y^2=9$ and converts to polar using x = r cos θ, y = r sin θ **M1**

(b) expands and uses $x^2+y^2 = r^2$ to obtain $r^2 = 6r \cos \theta$ **M1**

(b) divides by r (r ≠ 0) to get r = 6cos θ, cso **A1**

(b) Answer: r = 6cos(θ) (shown)

(c) uses area = integral from 0 to π/4 of (1/2) r^2 d θ with r = 6cos θ **M1**

(c) forms the integral (1/2) x integral of $36\cos^2(\theta)$ d θ = 18 x integral of $\cos^2(\theta)$ d θ **M1**

(c) uses $\cos^2(\theta) = (1+\cos(2\text{theta}))/2$ to integrate **M1**

(c) correct antiderivative 18[θ/2 + sin(2theta)/4] **A1**

(c) substitutes limits θ = π/4 and θ = 0 **M1**

(c) area = 9pi/4 + 9/2, oe exact form, cao **A1**

(c) Answer: Area = 9pi/4 + 9/2 (= (9/4)(π+2))

Question 6

(a) writes $\cos(5\text{theta}) + i \sin(5\text{theta}) = (\cos \theta + i \sin \theta)^5$ and expands using the binomial theorem **M1**

(a) identifies the real part: $\cos^5 \theta - 10\cos^3 \theta \sin^2 \theta + 5\cos \theta \sin^4 \theta$ **M1**

(a) substitutes $\sin^2 \theta = 1 - \cos^2 \theta$ throughout **M1**

(a) correctly simplifies and collects terms in cos θ **A1**

(a) $\cos(5\text{theta}) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$, cso **A1**

(a) Answer: $\cos(5\text{theta}) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$ (shown)

(b) uses part (a) to rewrite the equation as cos(5theta) = 0 **M1**

(b) sets 5theta = π/2 + k π **M1**

(b) θ = π/10, 3pi/10, π/2, 7pi/10, 9pi/10 (at least 3 correct) **A1**

(b) all five values correct within 0 < θ < π, cao **A1**

(b) Answer: θ = π/10, 3pi/10, π/2, 7pi/10, 9pi/10

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