1.1 Complex Numbers: 30-minute Pack

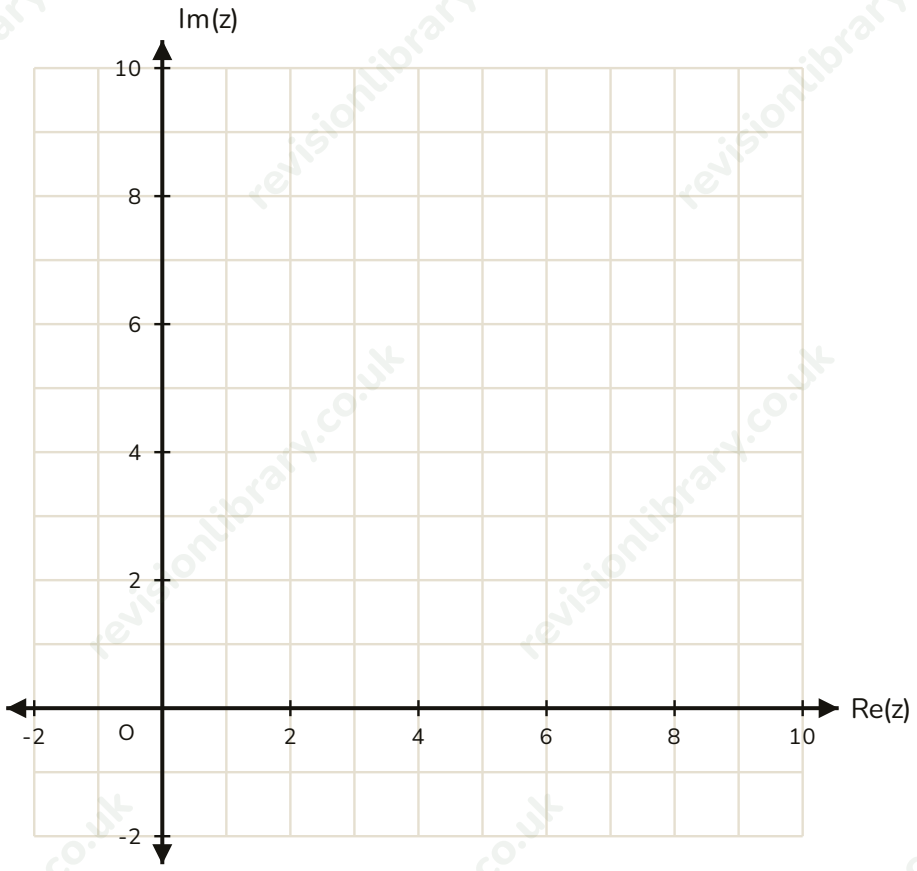
EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A locus C in the Argand diagram is defined by the equation $|z - (3 + 4i)| = 5$.



(a) Sketch the locus C, marking clearly the points where it intersects the real axis. **(3)**

(b) Find the greatest and least possible values of $|z|$ for points on C. **(3)**

(c) The point on C for which $\arg(z) = \pi/4$ (other than the origin) is $z = p + qi$. Find the values of p and q. **(4)**

(Total for Question 1 is 10 marks)

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2 Solve the equation $z^3 = -216$, giving each root in the form a + bi with a and b exact.

(a) Write -216 in modulus-argument form. **(2)**

(b) Hence find all 3 roots of the equation. **(4)**

(Total for Question 2 is 6 marks)

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3 The complex number z is z = -1 + i.

(a) Write z in modulus-argument form $r(\cos \theta + i \sin \theta)$, with θ in degrees. **(3)**

(b) Use de Moivre's theorem to find z^{11}, giving your answer in the form a + bi, with a and b exact. **(4)**

(Total for Question 3 is 7 marks)

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4 Solve the equation $z^3 = -27$, giving each root in the form a + bi with a and b exact.

(a) Write -27 in modulus-argument form. **(2)**

(b) Hence find all 3 roots of the equation. **(4)**

(Total for Question 4 is 6 marks)

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Mark scheme · 1.1 Complex Numbers: 30-minute Pack

Question 1

(a) sets z = x (real) and forms $(x-3)^2 + 16 = 25$ **M1**

(a) x = 0 and x = 6 found **A1**

(a) correct sketch: circle centre 3 + 4i, radius 5, passing through (0,0) and (6,0) **B1**

(a) Answer: Circle centre 3 + 4i, radius 5, crossing the real axis at z = 0 and z = 6

(b) finds distance from origin to centre = $\sqrt{3^2 + 4^2} = 5$ **M1**

(b) greatest |z| = 5 + 5 = 10 **A1**

(b) least |z| = 5 - 5 = 0, cao **A1**

(b) Answer: Greatest |z| = 10, least |z| = 0

(c) uses arg(z) = π/4 to set y = x, with x > 0 **M1**

(c) substitutes y = x into $(x-3)^2 + (x-4)^2 = 25$ **M1**

(c) solves to x = 0 (rejected, this is the origin) or x = 7 **A1**

(c) z = 7 + 7i, cao **A1**

(c) Answer: z = 7 + 7i (p = 7, q = 7)

Question 2

(a) modulus and argument **M1**

(a) 216(cos 180 + i sin 180) (plus multiples of 360 degrees) **A1**

(a) Answer: 216(cos 180 + i sin 180)

(b) modulus 6 and arguments (180 + 360k)/3 degrees for k = 0, 1, ..., 2 **M1**

(b) arguments 60, 180, 300 **A1**

(b) uses exact values of cosine and sine **M1**

(b) roots: $3 + 3\sqrt{3}\,i$; -6; $3 - 3\sqrt{3}\,i$ **A1**

(b) Answer: $3 + 3\sqrt{3}\,i$; -6; $3 - 3\sqrt{3}\,i$

Question 3

(a) finds the modulus **M1**

(a) $r = \sqrt{2}$ **A1**

(a) θ = 135 degrees **A1**

(a) Answer: $z = \sqrt{2}$ (cos 135 + i sin 135) degrees

(b) $z^{11} = (\sqrt{2})^{11}$ (cos(11 x 135) + i sin(11 x 135)) **M1**

(b) modulus $32\sqrt{2}$ and argument 1485 degrees **A1**

(b) uses exact values of cos(45) and sin(45) **M1**

(b) 32 + 32i **A1**

(b) Answer: 32 + 32i

Question 4

(a) modulus and argument **M1**

(a) 27(cos 180 + i sin 180) (plus multiples of 360 degrees) **A1**

(a) Answer: 27(cos 180 + i sin 180)

(b) modulus 3 and arguments (180 + 360k)/3 degrees for k = 0, 1, ..., 2 **M1**

(b) arguments 60, 180, 300 **A1**

(b) uses exact values of cosine and sine **M1**

(b) roots: $3/2 + (3/2)\sqrt{3}\,i$; -3; $3/2 - (3/2)\sqrt{3}\,i$ **A1**

(b) Answer: $3/2 + (3/2)\sqrt{3}\,i$; -3; $3/2 - (3/2)\sqrt{3}\,i$

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