1.1 Complex Numbers: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 70 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The complex number z satisfies the equation $2z^2 - 4z + 5 = 0$.

(a) Solve the equation, giving each root in the form a + bi. **(4)**

(b) For the root with positive imaginary part, find the modulus and the argument, giving the argument in radians to 3 significant figures. **(3)**

(c) Hence write this root in the form r(cos θ + i sin θ), where r and θ are given to 3 significant figures. **(2)**

(Total for Question 1 is 9 marks)

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2 The complex numbers z_1 and z_2 are given by $z_1 = -3 + 3i$ and $z_2 = -4 + 5i$.

(a) Find $z_1 z_2$ in the form a + bi. **(2)**

(b) Find z_1/z_2 in the form a + bi, where a and b are rational. **(3)**

(c) Find the two square roots of 12 + 16i, giving each in the form a + bi. **(4)**

(Total for Question 2 is 9 marks)

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3 The cubic equation $z^3 + pz^2 + qz - 90 = 0$, where p and q are real constants, has a root 3 + 3i.

(a) Write down another complex root of the equation, with a reason. **(2)**

(b) Find the third root of the equation. **(3)**

(c) Find the values of p and q. **(4)**

(Total for Question 3 is 9 marks)

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4 Prove by mathematical induction that, for all positive integers n, $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$.

(a) Show that the result is true for n = 1. **(1)**

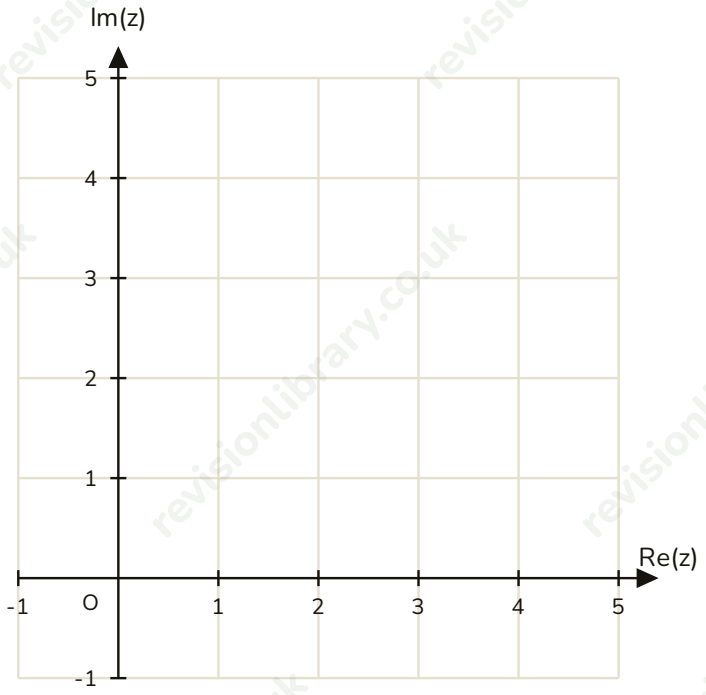
(b) Assuming the result is true for n = k, show that it is true for n = k + 1. **(4)**

(c) Hence complete the proof by induction, making clear reference to your results in parts (a) and (b). **(1)**

(Total for Question 4 is 6 marks)

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5 Loci C1 and C2 in the Argand diagram are defined by C1: $|z - 4| = |z - 2i|$, and C2: $\arg(z - 1) = \pi/4$.



(a) Show that C1 has cartesian equation $y = 2x - 3$. **(4)**

(b) State the cartesian equation of C2, including the restriction on x. **(2)**

(c) Find the point of intersection of C1 and C2, giving your answer in the form $a + bi$. **(3)**

(d) Find the exact area of the triangle formed by the point of intersection found in part (c), the point $z = 4$, and the origin. **(3)**

(Total for Question 5 is 12 marks)

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6 A transformation from the z-plane to the w-plane is defined by $w = (z - i)/(z + 1)$, where $z \neq -1$. The point z lies on the circle $|z| = 1$.

(a) Show that, as z varies on the circle $|z| = 1$, the image point $w = u + iv$ satisfies $v = -u$. **(7)**

(b) State the cartesian equation of the locus of w, and explain, with reference to the point $z = -1$, why this locus is a straight line rather than a circle. **(2)**

(Total for Question 6 is 9 marks)

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Mark scheme · 1.1 Complex Numbers: Test Pack

Question 1

(a) correct substitution into the quadratic formula, $z = [4 \pm \sqrt{16 - 40}] / 4$ **M1**

(a) discriminant simplified to $\sqrt{-24} = 2\sqrt{6}\ i$ (oe) **A1**

(a) simplifies to $z = 1 \pm (\sqrt{6}/2)\ i$ **M1**

(a) both roots correct, cao **A1**

(a) Answer: $z = 1 + (\sqrt{6}/2)\ i$ or $z = 1 - (\sqrt{6}/2)\ i$

(b) modulus = $\sqrt{1^2 + (\sqrt{6}/2)^2}$ **M1**

(b) modulus = $\sqrt{2.5}$, awrt 1.58 **A1**

(b) argument = $\arctan((\sqrt{6}/2) / 1)$, awrt 0.886 radians, cao **A1**

(b) Answer: $|z| = \sqrt{2.5}$ = awrt 1.58; arg(z) = awrt 0.886 radians

(c) uses the values of r and θ found in part (b) **M1**

(c) $z = 1.58(\cos 0.886 + i \sin 0.886)$, awrt **A1**

(c) Answer: $z = 1.58 (\cos(0.886) + i \sin(0.886))$

Question 2

(a) expands $(a + bi)(c + di)$ with $i^2 = -1$ **M1**

(a) $-3 - 27i$ **A1**

(a) Answer: $-3 - 27i$

(b) multiplies numerator and denominator by the conjugate of z_2 **M1**

(b) denominator 41 **A1**

(b) $27/41 + (3/41)i$ **A1**

(b) Answer: $27/41 + (3/41)i$

(c) sets $(x + iy)^2 = 12 + 16i$, so $x^2 - y^2 = 12$ and $2xy = 16$ **M1**

(c) eliminates y to obtain a quadratic in x^2 (or finds a pair by inspection) **M1**

(c) $x = 4$, $y = 2$ **A1**

(c) $4 + 2i$ and $-4 - 2i$ **A1**

(c) Answer: $4 + 2i$ and $-4 - 2i$

Question 3

(a) $3 - 3i$ **B1**

(a) complex roots of a polynomial with real coefficients occur in conjugate pairs **B1**

(a) Answer: $3 - 3i$

(b) uses the product of the roots: $(\alpha)(\alpha^*)(r) = -(-90)$ **M1**

(b) $((3)^2 + 3^2)\ r = 90$ **A1**

(b) $r = 5$ **A1**

(b) Answer: 5

(c) uses sum of roots = $-p$ **M1**

(c) $p = -11$ **A1**

(c) uses the sum of products of pairs = q **M1**

(c) $q = 48$ **A1**

(c) Answer: $p = -11$, $q = 48$

Question 4

(a) LHS = $\cos \theta + i \sin \theta$ = RHS when n = 1, so the result is true for n = 1 **B1**

(a) Answer: True for n = 1, since both sides equal $\cos \theta + i \sin \theta$

(b) writes $(\cos \theta + i \sin \theta)^{k+1} = (\cos \theta + i \sin \theta)^k (\cos \theta + i \sin \theta)$ and substitutes the assumption $\cos(k \theta) + i \sin(k \theta)$ **M1**

(b) expands the product: $[\cos(k \theta)\cos \theta - \sin(k \theta)\sin \theta] + i[\sin(k \theta)\cos \theta + \cos(k \theta)\sin \theta]$ **M1**

(b) uses the addition formulae to write the real part as $\cos(k \theta + \theta)$ **A1**

(b) writes the imaginary part as $\sin(k \theta + \theta)$, giving $\cos((k+1)\theta) + i \sin((k+1)\theta)$, cso **A1**

(b) Answer: $(\cos \theta + i \sin \theta)^{k+1} = \cos((k+1)\theta) + i \sin((k+1)\theta)$ (shown)

(c) correct conclusion: since the result is true for n = 1, and true for n = k implies true for n = k+1, the result is true for all positive integers n by mathematical induction **B1**

(c) Answer: True for all positive integers n by mathematical induction

Question 5

(a) writes $z = x + iy$ and squares both sides: $(x-4)^2 + y^2 = x^2 + (y-2)^2$ **M1**

(a) expands both sides correctly **M1**

(a) simplifies to $-8x + 16 = -4y + 4$ **A1**

(a) rearranges to $y = 2x - 3$, cso **A1**

(a) Answer: $y = 2x - 3$ (shown)

(b) $y = x - 1$ **B1**

(b) restriction $x > 1$ (since arg = π/4 requires both $x - 1 > 0$ and $y > 0$) **B1**

(b) Answer: $y = x - 1$, for $x > 1$

(c) solves $2x - 3 = x - 1$ simultaneously **M1**

(c) $x = 2$, $y = 1$ **A1**

(c) checks $x = 2 > 1$ is valid; $z = 2 + i$, cao **A1**

(c) Answer: $z = 2 + i$

(d) uses area = $(1/2)|x1(y2-y3)+x2(y3-y1)+x3(y1-y2)|$ with vertices (2,1), (4,0), (0,0) **M1**

(d) correct substitution: $(1/2)|2(0-0)+4(0-1)+0(1-0)|$ **A1**

(d) area = 2 square units, cao **A1**

(d) Answer: Area = 2 square units

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Question 6

(a) lets $z = x + iy$ with $x^2+y^2=1$ and writes $w = (x + i(y-1)) / ((x+1) + iy)$ **M1**

(a) multiplies numerator and denominator by the conjugate $(x+1) - iy$ **M1**

(a) simplifies the denominator to $(x+1)^2 + y^2 = 2(x+1)$ using $x^2+y^2=1$ **M1**

(a) expands the numerator product $[x + i(y-1)][(x+1) - iy]$ **M1**

(a) real part simplifies, using $x^2+y^2=1$, to $1 + x - y$ **A1**

(a) imaginary part simplifies to $y - x - 1$ **A1**

(a) since $y - x - 1 = -(1+x-y)$, concludes $v = -u$, cso **A1**

(a) Answer: v = -u (shown), i.e. Re(w) + Im(w) = 0

(b) equation $u + v = 0$ (or $v = -u$) stated **B1**

(b) explains that $z = -1$ lies on the circle $|z|=1$, and this point is mapped to infinity by $w = (z-i)/(z+1)$ since the denominator is zero there; a circle passing through the pole of a Mobius transformation maps to a straight line **B1**

(b) Answer: u + v = 0; because z = -1 lies on |z| = 1 and is the pole of the transformation (denominator zero there), so its image is the point at infinity, meaning the image of the circle is a straight line

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