1.1 Complex Numbers

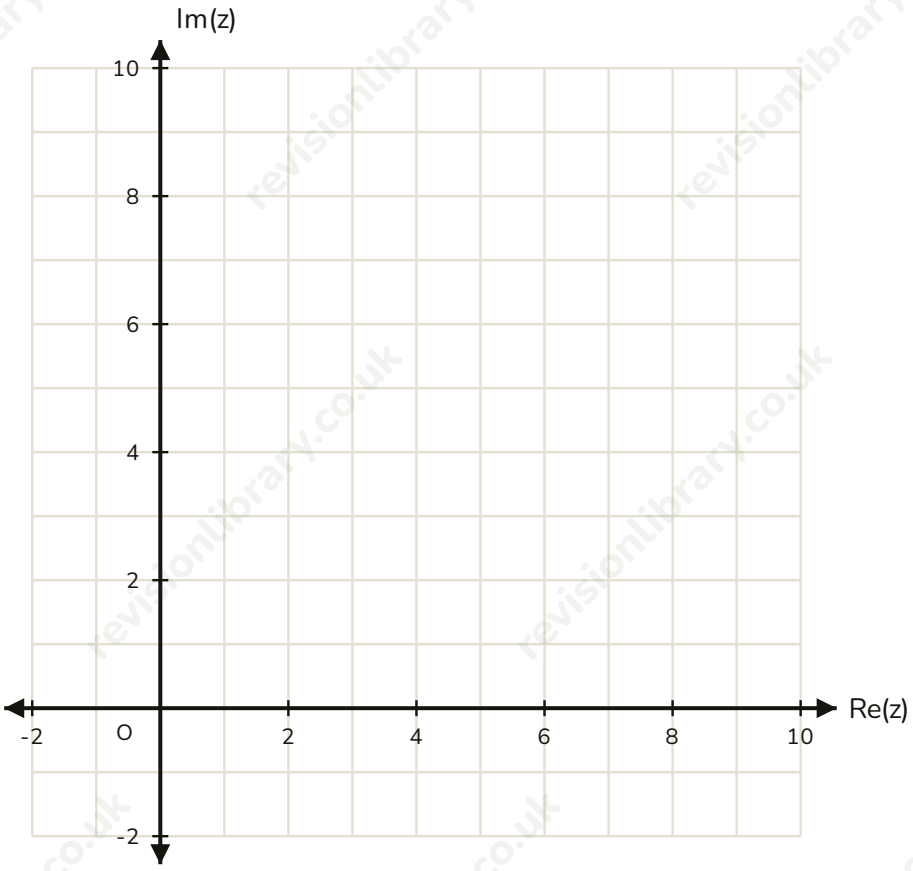
EDEXCEL 9FM0 · Calculator allowed · about 215 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A locus C in the Argand diagram is defined by the equation $|z - (3 + 4i)| = 5$.



(a) Sketch the locus C, marking clearly the points where it intersects the real axis. **(3)**

(b) Find the greatest and least possible values of $|z|$ for points on C. **(3)**

(c) The point on C for which $\arg(z) = \pi/4$ (other than the origin) is $z = p + qi$. Find the values of p and q. **(4)**

(Total for Question 1 is 10 marks)

2 The complex number z is $z = 1 + i$.

(a) Write z in modulus-argument form $r(\cos \theta + i \sin \theta)$, with θ in degrees. **(3)**

(b) Use de Moivre's theorem to find z^3, giving your answer in the form $a + bi$, with a and b exact. **(4)**

(Total for Question 2 is 7 marks)

3 Solve the equation $z^3 = -216$, giving each root in the form $a + bi$ with a and b exact.

(a) Write -216 in modulus-argument form. **(2)**

(b) Hence find all 3 roots of the equation. **(4)**

(Total for Question 3 is 6 marks)

4 The complex number z is $z = -1 + i$.

(a) Write z in modulus-argument form $r(\cos \theta + i \sin \theta)$, with θ in degrees. **(3)**

(b) Use de Moivre's theorem to find z^{11}, giving your answer in the form $a + bi$, with a and b exact. **(4)**

(Total for Question 4 is 7 marks)

5 Solve the equation $z^3 = -27$, giving each root in the form a + bi with a and b exact.

(a) Write -27 in modulus-argument form. **(2)**

(b) Hence find all 3 roots of the equation. **(4)**

(Total for Question 5 is 6 marks)

6 The complex number z satisfies $|z - (3 - 2i)| = 4$.

(a) Describe the locus of z geometrically, and write its equation in Cartesian form. **(4)**

(b) Find the greatest value of $|z|$ and the least value of $|z|$. **(3)**

(Total for Question 6 is 7 marks)

7 The complex number z is $z = 1 - \sqrt{3}i$.

(a) Write z in modulus-argument form $r(\cos\theta + i\sin\theta)$, with θ in degrees. **(3)**

(b) Use de Moivre's theorem to find z^9, giving your answer in the form $a + bi$, with a and b exact. **(4)**

(Total for Question 7 is 7 marks)

8 The cubic equation $z^3 + pz^2 + qz - 17 = 0$, where p and q are real constants, has a root $-4 + i$.

(a) Write down another complex root of the equation, with a reason. **(2)**

(b) Find the third root of the equation. **(3)**

(c) Find the values of p and q. **(4)**

(Total for Question 8 is 9 marks)

9 The complex number z satisfies $|z - (5 + 3i)| = 4$.

(a) Describe the locus of z geometrically, and write its equation in Cartesian form. **(4)**

(b) Find the greatest value of $|z|$ and the least value of $|z|$. **(3)**

(Total for Question 9 is 7 marks)

10 The complex numbers z_1 and z_2 are $z_1 = 7 - 3i$ and $z_2 = 6 + 4i$.

(a) Find the modulus and the argument of z_1, giving the argument in radians to 3 significant figures, in the range $-\pi < \arg \leq \pi$. **(4)**

(b) Find the modulus and argument of z_2 to 3 significant figures. **(3)**

(c) Hence find the modulus and argument of $z_1 z_2$ and of z_1/z_2, to 3 significant figures, with arguments in the range $-\pi < \arg \leq \pi$. **(3)**

(d) Find the modulus and argument of $(z_1)^4$ to 3 significant figures, with the argument in the range $-\pi < \arg \leq \pi$. **(3)**

(Total for Question 10 is 13 marks)

11 The complex numbers z_1 and z_2 are given by $z_1 = -1 + 4i$ and $z_2 = 5 - 2i$.

(a) Find $z_1 z_2$ in the form $a + bi$. **(2)**

(b) Find z_1/z_2 in the form $a + bi$, where a and b are rational. **(3)**

(c) Find the two square roots of $-9 + 40i$, giving each in the form $a + bi$. **(4)**

(Total for Question 11 is 9 marks)

12 The complex numbers z_1 and z_2 are $z_1 = -5 - i$ and $z_2 = -1 + 2i$.

(a) Find the modulus and the argument of z_1, giving the argument in radians to 3 significant figures, in the range $-\pi < \arg \leq \pi$. **(4)**

(b) Find the modulus and argument of z_2 to 3 significant figures. **(3)**

(c) Hence find the modulus and argument of $z_1 z_2$ and of z_1/z_2, to 3 significant figures, with arguments in the range $-\pi < \arg \leq \pi$. **(3)**

(d) Find the modulus and argument of $(z_1)^3$ to 3 significant figures, with the argument in the range $-\pi < \arg \leq \pi$. **(3)**

(Total for Question 12 is 13 marks)

13 This question is about De Moivre's theorem.

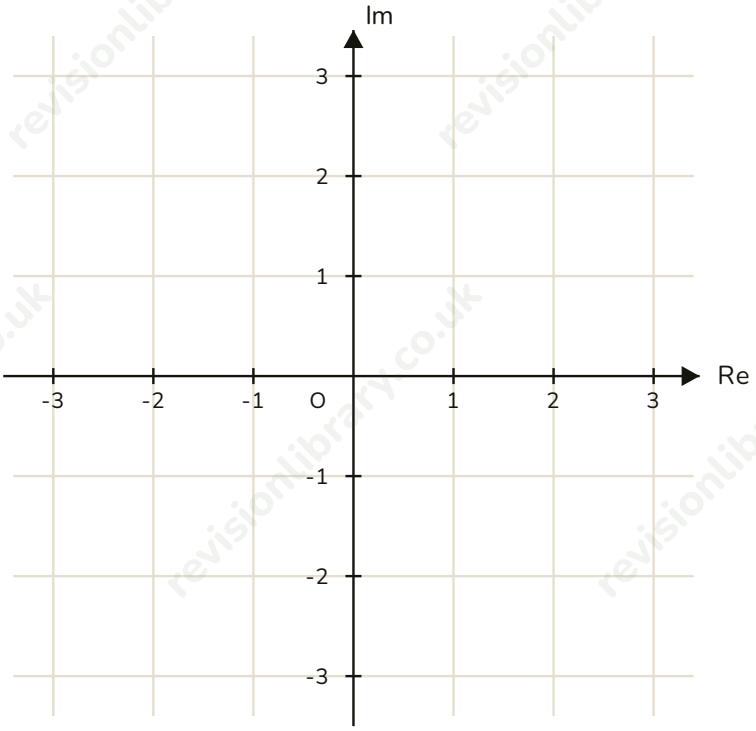
(a) State De Moivre's theorem for a positive integer n. **(1)**

(b) Hence find, in the form a + bi, the value of $(\cos(\pi/6) + i \sin(\pi/6))^9$. **(3)**

(c) Find the smallest positive integer n for which $(\cos(\pi/6) + i \sin(\pi/6))^n$ is a positive real number. **(3)**

(Total for Question 13 is 7 marks)

14 The complex number $w = -8i$.



(a) Find the three cube roots of w, giving each root in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$. **(5)**

(b) Show that the sum of the three cube roots is 0. **(3)**

(c) Plot the three roots on an Argand diagram and state, with a reason, the geometric shape they form. **(2)**

(Total for Question 14 is 10 marks)

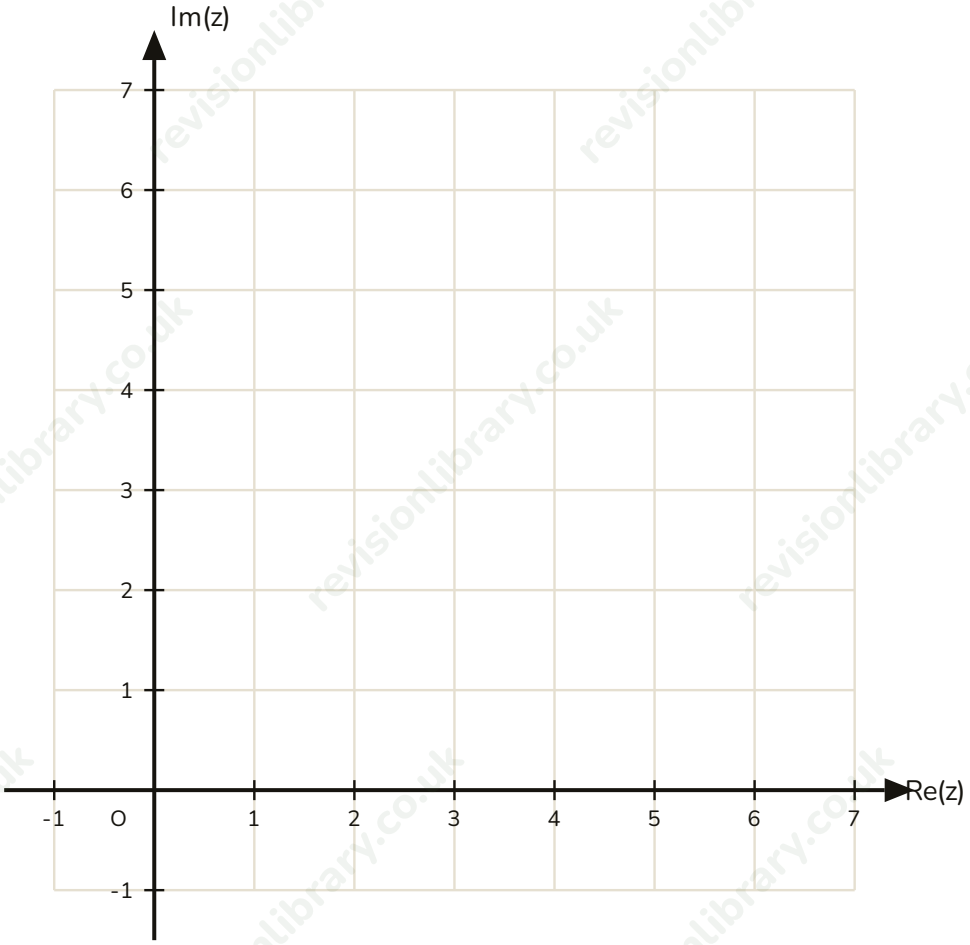
15 $z = 2 - i$ is one root of the cubic equation $z^3 - 6z^2 + 13z - 10 = 0$.

(a) Explain why $z = 2 + i$ must also be a root of this equation, and hence find the third root. **(5)**

(b) State the sum and the product of all three roots of the equation, and verify that these agree with the coefficients of the cubic. **(4)**

(Total for Question 15 is 9 marks)

16 The region R in the Argand diagram is defined by $|z - 3| \leq 3$ and $0 \leq \arg(z) \leq \pi/4$.



(a) Sketch the region R on an Argand diagram. **(3)**

(b) Show that the circle $|z - 3| = 3$ has polar equation $r = 6\cos(\theta)$, for $-\pi/2 \leq \theta \leq \pi/2$. **(3)**

(c) Hence find the exact area of the region R. **(6)**

(Total for Question 16 is 12 marks)

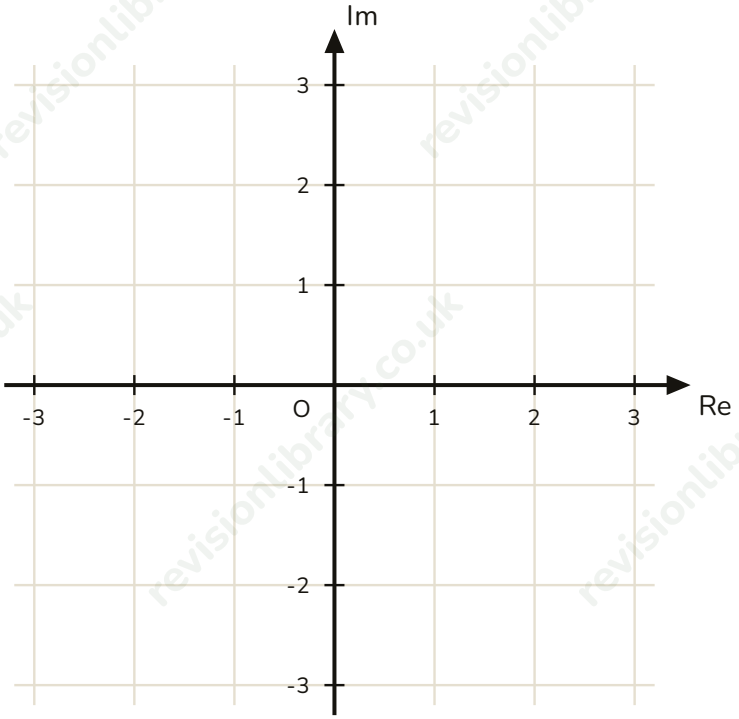
17 This question uses De Moivre's theorem to derive a multiple-angle identity.

(a) Use De Moivre's theorem and the binomial expansion of $(\cos\theta + i\sin\theta)^5$ to show that $\cos(5\theta) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$. **(5)**

(b) Hence solve, for $0 < \theta < \pi$, the equation $16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta) = 0$, giving each answer as an exact multiple of π. **(4)**

(Total for Question 17 is 9 marks)

18 The complex number w satisfies $w^6 = 64$.



(a) Find all six roots of this equation, giving each in the form $r(\cos\theta + i\sin\theta)$, with $-\pi < \theta \leq \pi$. **(6)**

(b) Show that these six roots form the vertices of a regular hexagon in the Argand diagram, and find its exact perimeter. **(3)**

(c) Find the exact area enclosed by the hexagon. **(3)**

(Total for Question 18 is 12 marks)

19 Let $z = \cos\theta + i\sin\theta$.

(a) Show that $\sin^4(\theta)$ can be written in the form Acos(4theta) + Bcos(2theta) + C, where A, B and C are constants to be found. **(5)**

(b) Hence find the exact value of the integral from 0 to $\pi/2$ of $\sin^4(\theta)\, d\theta$. **(4)**

(Total for Question 19 is 9 marks)

Mark scheme · 1.1 Complex Numbers

Question 1

(a) sets z = x (real) and forms $(x-3)^2 + 16 = 25$ **M1**

(a) x = 0 and x = 6 found **A1**

(a) correct sketch: circle centre 3 + 4i, radius 5, passing through (0,0) and (6,0) **B1**

(a) Answer: Circle centre 3 + 4i, radius 5, crossing the real axis at z = 0 and z = 6

(b) finds distance from origin to centre = $\sqrt{3^2 + 4^2} = 5$ **M1**

(b) greatest |z| = 5 + 5 = 10 **A1**

(b) least |z| = 5 - 5 = 0, cao **A1**

(b) Answer: Greatest |z| = 10, least |z| = 0

(c) uses arg(z) = π/4 to set y = x, with x > 0 **M1**

(c) substitutes y = x into $(x-3)^2 + (x-4)^2 = 25$ **M1**

(c) solves to x = 0 (rejected, this is the origin) or x = 7 **A1**

(c) z = 7 + 7i, cao **A1**

(c) Answer: z = 7 + 7i (p = 7, q = 7)

Question 2

(a) finds the modulus **M1**

(a) r = $\sqrt{2}$ **A1**

(a) θ = 45 degrees **A1**

(a) Answer: z = $\sqrt{2}$ (cos 45 + i sin 45) degrees

(b) $z^3 = (\sqrt{2})^3$ (cos(3 x 45) + i sin(3 x 45)) **M1**

(b) modulus $2\sqrt{2}$ and argument 135 degrees **A1**

(b) uses exact values of cos(135) and sin(135) **M1**

(b) -2 + 2i **A1**

(b) Answer: -2 + 2i

Question 3

(a) modulus and argument **M1**

(a) 216(cos 180 + i sin 180) (plus multiples of 360 degrees) **A1**

(a) Answer: 216(cos 180 + i sin 180)

(b) modulus 6 and arguments (180 + 360k)/3 degrees for k = 0, 1, ..., 2 **M1**

(b) arguments 60, 180, 300 **A1**

(b) uses exact values of cosine and sine **M1**

(b) roots: $3 + 3\sqrt{3}$ i; -6; $3 - 3\sqrt{3}$ i **A1**

(b) Answer: $3 + 3\sqrt{3}$ i; -6; $3 - 3\sqrt{3}$ i

Question 4

(a) finds the modulus **M1**

(a) r = $\sqrt{2}$ **A1**

(a) θ = 135 degrees **A1**

(a) Answer: z = $\sqrt{2}$ (cos 135 + i sin 135) degrees

(b) $z^{11} = (\sqrt{2})^{11}$ (cos(11 x 135) + i sin(11 x 135)) **M1**

(b) modulus $32\sqrt{2}$ and argument 1485 degrees **A1**

(b) uses exact values of cos(45) and sin(45) **M1**

(b) 32 + 32i **A1**

(b) Answer: 32 + 32i

Question 5

(a) modulus and argument **M1**

(a) 27(cos 180 + i sin 180) (plus multiples of 360 degrees) **A1**

(a) Answer: 27(cos 180 + i sin 180)

(b) modulus 3 and arguments (180 + 360k)/3 degrees for k = 0, 1, ..., 2 **M1**

(b) arguments 60, 180, 300 **A1**

(b) uses exact values of cosine and sine **M1**

(b) roots: $3/2 + (3/2)\sqrt{3}$ i; -3; $3/2 - (3/2)\sqrt{3}$ i **A1**

(b) Answer: $3/2 + (3/2)\sqrt{3}$ i; -3; $3/2 - (3/2)\sqrt{3}$ i

Question 6

(a) a circle with centre (3, -2) **B1**

(a) radius 4 **B1**

(a) writes z = x + iy and uses $|z - w|^2$ **M1**

(a) $(x - 3)^2 + (y + 2)^2 = 16$ **A1**

(a) Answer: $(x - 3)^2 + (y + 2)^2 = 16$

(b) distance of the centre from the origin = $\sqrt{13}$ **M1**

(b) greatest = $\sqrt{13} + 4$ **A1**

(b) least = $|\sqrt{13} - 4|$ (the origin lies inside the circle when $\sqrt{13} < 4$) **A1**

(b) Answer: greatest $\sqrt{13} + 4 = 7.606$, least 0.3944

Question 7

(a) finds the modulus **M1**

(a) r = 2 **A1**

(a) θ = -60 degrees (accept 300 degrees) **A1**

(a) Answer: z = 2(cos -60 + i sin -60) degrees (or 2(cos 300 + i sin 300))

(b) $z^9 = (2)^9$ (cos(9 x -60) + i sin(9 x -60)) **M1**

(b) modulus 512 and argument -540 degrees (or an equivalent argument, e.g. 2700 or 180 degrees) **A1**

(b) uses exact values of cos(180) and sin(180) **M1**

(b) -512 **A1**

(b) Answer: -512

Question 8

(a) -4 - i **B1**

(a) complex roots of a polynomial with real coefficients occur in conjugate pairs **B1**

(a) Answer: -4 - i

(b) uses the product of the roots: (α)(α*)(r) = -(-17) **M1**

(b) $((-4)^2 + 1^2)$ r = 17 **A1**

(b) r = 1 **A1**

(b) Answer: 1

(c) uses sum of roots = -p **M1**

(c) p = 7 **A1**

(c) uses the sum of products of pairs = q **M1**

(c) q = 9 **A1**

(c) Answer: p = 7, q = 9

Question 9

(a) a circle with centre (5, 3) **B1**

(a) radius 4 **B1**

(a) writes $z = x + iy$ and uses $|z - w|^2$ **M1**

(a) $(x - 5)^2 + (y - 3)^2 = 16$ **A1**

(a) Answer: $(x - 5)^2 + (y - 3)^2 = 16$

(b) distance of the centre from the origin = $\sqrt{34}$ **M1**

(b) greatest = $\sqrt{34} + 4$ **A1**

(b) least = $|\sqrt{34} - 4|$ **A1**

(b) Answer: greatest $\sqrt{34} + 4 = 9.831$, least 1.831

Question 10

(a) modulus = $\sqrt{(7)^2 + (-3)^2}$ **M1**

(a) $|z_1| = \sqrt{58} = 7.62$ **A1**

(a) uses the inverse tangent with a quadrant check **M1**

(a) arg $z_1 = -0.405$ **A1**

(a) Answer: $|z_1| = \sqrt{58}$, arg $z_1 = -0.405$

(b) modulus and argument by the same method **M1**

(b) $|z_2| = 7.21$ **A1**

(b) arg $z_2 = 0.588$ **A1**

(b) Answer: $|z_2| = 7.21$, arg $z_2 = 0.588$

(c) $|z_1 z_2| = |z_1||z_2|$ and $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ (adjusted to the range) **M1**

(c) $|z_1 z_2| = 54.9$, arg = 0.183 **A1**

(c) $|z_1/z_2| = 1.06$, arg = -0.993 **A1**

(c) Answer: $z_1 z_2$: modulus 54.9, arg 0.183; z_1/z_2: modulus 1.06, arg -0.993

(d) uses de Moivre: modulus to the power n, argument multiplied by n **M1**

(d) modulus 3360 **A1**

(d) arg = -1.62 **A1**

(d) Answer: modulus 3360, arg -1.62

Question 11

(a) expands $(a + bi)(c + di)$ with $i^2 = -1$ **M1**

(a) $3 + 22i$ **A1**

(a) Answer: $3 + 22i$

(b) multiplies numerator and denominator by the conjugate of z_2 **M1**

(b) denominator 29 **A1**

(b) $-13/29 + (18/29)i$ **A1**

(b) Answer: $-13/29 + (18/29)i$

(c) sets $(x + iy)^2 = -9 + 40i$, so $x^2 - y^2 = -9$ and $2xy = 40$ **M1**

(c) eliminates y to obtain a quadratic in x^2 (or finds a pair by inspection) **M1**

(c) x = 4, y = 5 **A1**

(c) 4 + 5i and -4 - 5i **A1**

(c) Answer: 4 + 5i and -4 - 5i

Question 12

(a) modulus = $\sqrt{(-5)^2 + (-1)^2}$ **M1**

(a) $|z_1| = \sqrt{26} = 5.10$ **A1**

(a) uses the inverse tangent with a quadrant check **M1**

(a) arg $z_1 = -2.94$ **A1**

(a) Answer: $|z_1| = \sqrt{26}$, arg $z_1 = -2.94$

(b) modulus and argument by the same method **M1**

(b) $|z_2| = 2.24$ **A1**

(b) arg $z_2 = 2.03$ **A1**

(b) Answer: $|z_2| = 2.24$, arg $z_2 = 2.03$

(c) $|z_1 z_2| = |z_1||z_2|$ and $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ (adjusted to the range) **M1**

(c) $|z_1 z_2| = 11.4$, arg = -0.910 **A1**

(c) $|z_1/z_2| = 2.28$, arg = 1.30 **A1**

(c) Answer: $z_1 z_2$: modulus 11.4, arg -0.910; z_1/z_2: modulus 2.28, arg 1.30

(d) uses de Moivre: modulus to the power n, argument multiplied by n **M1**

(d) modulus 133 **A1**

(d) arg = -2.55 **A1**

(d) Answer: modulus 133, arg -2.55

Question 13

(a) $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$ **B1**

(a) Answer: $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

(b) uses De Moivre with $n\theta = 9 \times \pi/6 = 3\pi/2$ **M1**

(b) $\cos(3\pi/2) = 0$ **A1**

(b) $\sin(3\pi/2) = -1$, giving answer -i, cao **A1**

(b) Answer: -i

(c) requires $n \times \pi/6 = 2k\pi$ for a positive integer k **M1**

(c) n = 12k **A1**

(c) smallest positive integer n = 12, cao **A1**

(c) Answer: n = 12

Question 14

(a) writes $w = 8(\cos(-\pi/2) + i\sin(-\pi/2))$ **M1**

(a) finds the modulus of each root, $r = 8^{1/3} = 2$ **M1**

(a) uses $\theta = (-\pi/2 + 2k\pi)/3$ for k = -1, 0, 1 **M1**

(a) two of the three roots correct **A1**

(a) all three roots correct: $2(\cos(-\pi/6)+i\sin(-\pi/6))$, $2(\cos(\pi/2)+i\sin(\pi/2))$, $2(\cos(-5\pi/6)+i\sin(-5\pi/6))$ **A1**

(a) Answer: $2(\cos(-\pi/6)+i\sin(-\pi/6))$, $2(\cos(\pi/2)+i\sin(\pi/2))$ [= 2i], $2(\cos(-5\pi/6)+i\sin(-5\pi/6))$

(b) converts each root to cartesian form: $\sqrt{3} - i$, 2i, $-\sqrt{3} - i$ **M1**

(b) all three cartesian forms correct **A1**

(b) sums to $(\sqrt{3} - \sqrt{3}) + i(-1+2-1) = 0$, cso **A1**

(b) Answer: 0 (shown)

(c) three points plotted correctly, each at distance 2 from the origin, at arguments $-\pi/6$, $\pi/2$ and $-5\pi/6$ **B1**

(c) states an equilateral triangle, since the roots are equally spaced by $2\pi/3$ around a circle of radius 2 **B1**

(c) Answer: Equilateral triangle inscribed in a circle of radius 2, centre the origin

Question 15

(a) states that since the coefficients of the cubic are all real, complex roots occur in conjugate pairs, so z = 2 + i is also a root **B1**

(a) forms the quadratic factor (z-(2-i))(z-(2+i)) = z^2 - 4z + 5 **M1**

(a) divides z^3 - $6z^2$ + 13z - 10 by z^2 - 4z + 5 **M1**

(a) quotient z - 2 **A1**

(a) third root z = 2, cao **A1**

(a) Answer: z = 2 + i (conjugate root); third root z = 2

(b) sum of roots = (2-i)+(2+i)+2 = 6 **B1**

(b) product of roots = (2-i)(2+i)(2) = 5 x 2 = 10 **B1**

(b) compares with -(coefficient of z^2)/(coefficient of z^3) = 6 and -(constant term)/(coefficient of z^3) = 10 **M1**

(b) both agree, so verified, cso **A1**

(b) Answer: Sum = 6, product = 10, both consistent with the coefficients of the cubic

Question 16

(a) circle centre 3, radius 3, passing through the origin, drawn correctly **B1**

(a) half-lines arg(z) = 0 and arg(z) = π/4 drawn from the origin **B1**

(a) region R (bounded by the circle and lying between the two half-lines) correctly identified/shaded **B1**

(a) Answer: Region bounded by the circle |z-3|=3 and the half-lines arg(z)=0, arg(z)=π/4, with the origin on the boundary

(b) writes |z-3|=3 as $(x-3)^2+y^2=9$ and converts to polar using x = r cos θ, y = r sin θ **M1**

(b) expands and uses $x^2+y^2 = r^2$ to obtain r^2 = 6r cos θ **M1**

(b) divides by r (r ≠ 0) to get r = 6cos θ, cso **A1**

(b) Answer: r = 6cos(θ) (shown)

(c) uses area = integral from 0 to π/4 of (1/2) r^2 d θ with r = 6cos θ **M1**

(c) forms the integral (1/2) x integral of $36\cos^2(\theta)$ d θ = 18 x integral of $\cos^2(\theta)$ d θ **M1**

(c) uses $\cos^2(\theta) = (1+\cos(2\theta))/2$ to integrate **M1**

(c) correct antiderivative 18[θ/2 + sin(2theta)/4] **A1**

(c) substitutes limits θ = π/4 and θ = 0 **M1**

(c) area = 9pi/4 + 9/2, oe exact form, cao **A1**

(c) Answer: Area = 9pi/4 + 9/2 (= (9/4)(π+2))

Question 17

(a) writes cos(5theta) + i sin(5theta) = $(\cos\theta + i\sin\theta)^5$ and expands using the binomial theorem **M1**

(a) identifies the real part: $\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ **M1**

(a) substitutes $\sin^2\theta = 1 - \cos^2\theta$ throughout **M1**

(a) correctly simplifies and collects terms in cos θ **A1**

(a) $\cos(5\theta) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$, cso **A1**

(a) Answer: $\cos(5\theta) = 16\cos^5(\theta) - 20\cos^3(\theta) + 5\cos(\theta)$ (shown)

(b) uses part (a) to rewrite the equation as cos(5theta) = 0 **M1**

(b) sets 5theta = π/2 + k π **M1**

(b) θ = π/10, 3pi/10, π/2, 7pi/10, 9pi/10 (at least 3 correct) **A1**

(b) all five values correct within 0 < θ < π, cao **A1**

(b) Answer: θ = π/10, 3pi/10, π/2, 7pi/10, 9pi/10

Question 18

(a) writes 64 = 64(cos 0 + i sin 0) **M1**

(a) modulus of each root r = $64^{1/6}$ = 2 **A1**

(a) uses θ = 2k π/6 = k π/3 for six consecutive integers k **M1**

(a) arguments 0, π/3, 2pi/3 correct **A1**

(a) arguments π, -2pi/3, -π/3 correct **A1**

(a) all six roots written correctly in the required form, cao **A1**

(a) Answer: 2(cos0+isin0), 2(cos(π/3)+isin(π/3)), 2(cos(2pi/3)+isin(2pi/3)), 2(cos(π)+isin(π)), 2(cos(-2pi/3)+isin(-2pi/3)), 2(cos(-π/3)+isin(-π/3))

(b) all six roots have modulus 2, so all lie on a circle of radius 2 centred at the origin, spaced at equal angles of π/3, forming a regular hexagon **B1**

(b) finds the side length using 2 x 2 x sin(π/6) **M1**

(b) perimeter = 6 x 2 = 12, cao **A1**

(b) Answer: Perimeter = 12

(c) uses area = 6 x (1/2) r^2 sin(π/3) **M1**

(c) substitutes r = 2: 6 x (1/2)(4)sin(π/3) **A1**

(c) area = $6\sqrt{3}$, cao **A1**

(c) Answer: $6\sqrt{3}$

Question 19

(a) uses 2i sin(θ) = z - 1/z **M1**

(a) expands $(z - 1/z)^4$ using the binomial theorem **M1**

(a) uses $z^k + 1/z^k = 2\cos(k\theta)$ to group terms **M1**

(a) correctly simplifies to $(2i\sin\theta)^4 = 2\cos(4\theta) - 8\cos(2\theta) + 6$ **A1**

(a) divides by 16 to give $\sin^4(\theta) = (1/8)\cos(4\theta) - (1/2)\cos(2\theta) + 3/8$, so A=1/8, B=-1/2, C=3/8, cso **A1**

(a) Answer: $\sin^4(\theta) = (1/8)\cos(4\theta) - (1/2)\cos(2\theta) + 3/8$ (A=1/8, B=-1/2, C=3/8)

(b) integrates term by term: (1/32)sin(4theta) - (1/4)sin(2theta) + (3/8)θ **M1**

(b) substitutes limits θ = π/2 and θ = 0 **M1**

(b) evaluates the sin(4theta) and sin(2theta) terms to 0 at both limits **A1**

(b) exact value = 3pi/16, cao **A1**

(b) Answer: 3pi/16