1.10 Series and Summation: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Use the method of differences, or standard results, to answer this question.

(a) Show that the sum from r = 1 to n of $5r(r + 1)(r + 2) = 5n(n + 1)(n + 2)(n + 3)/4$. **(3)**

(b) Hence evaluate the sum from r = 4 to 18 of $5r(r + 1)(r + 2)$. **(2)**

(Total for Question 1 is 5 marks)

2 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(2r^2 - 5r + 0)$ can be written as $n(4n^2 - 9n - 13)/6$. **(4)**

(b) Hence evaluate the sum from r = 5 to r = 8 of $(2r^2 - 5r + 0)$. **(4)**

(Total for Question 2 is 8 marks)

3 Use the standard results for the sums of r^2 and r^3 to answer this question.

(a) Show that the sum from r = 1 to n of $(6r^3 - 3r^2)$ is $n(n + 1)(18n^2 + 6n - 6)/12$. **(3)**

(b) Hence find the value of the sum from r = 4 to r = 11 of $(6r^3 - 3r^2)$. **(3)**

(Total for Question 3 is 6 marks)

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4 This question is about the series with r-th term $u_r = 2/(r(r + 4))$.

(a) Show that $2/(r(r + 4)) = (1/2)(1/r - 1/(r + 4))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 17. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 4 is 8 marks)

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5 Use the standard results for the sums of r^2 and r^3 to answer this question.

(a) Show that the sum from r = 1 to n of $(6r^3 + 3r^2)$ is $n(n + 1)(18n^2 + 30n + 6)/12$. **(3)**

(b) Hence find the value of the sum from r = 5 to r = 11 of $(6r^3 + 3r^2)$. **(3)**

(Total for Question 5 is 6 marks)

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6 For a positive integer n, consider $\sum_{r=n+1}^{2n} r^2$, the sum of the squares of the integers from n+1 to 2n.

(a) Show that $\sum_{r=n+1}^{2n} r^2 = n(2n+1)(7n+1)/6$. **(5)**

(b) Hence find $\sum_{r=6}^{10} r^2$ by using the result in part (a) with n = 5. **(2)**

(Total for Question 6 is 7 marks)

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7 This question derives the standard result for $\sum_{r=1}^{n} r^2$ from first principles, using the method of differences.

(a) Show that $r^3 - (r-1)^3 = 3r^2 - 3r + 1$. **(2)**

(b) By summing both sides of the identity in part (a) from r = 1 to n, and using the method of differences on the left-hand side, show that $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$. **(5)**

(You may use $\sum_{r=1}^{n} r = n(n+1)/2$ without proof.)

(Total for Question 7 is 7 marks)

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Mark scheme · 1.10 Series and Summation: 1-hour Pack

Question 1

(a) uses the identity $r(r + 1)(r + 2) = [r(r + 1)(r + 2)(r + 3) - (r - 1)r(r + 1)(r + 2)]/4$ **M1**

(a) applies the method of differences so that intermediate terms cancel **M1**

(a) obtains $n(n + 1)(n + 2)(n + 3)/4$ (cso) **A1**

(a) Answer: $5n(n + 1)(n + 2)(n + 3)/4$

(b) S(18) - S(3) **M1**

(b) 179100 **A1**

(b) Answer: 179100

Question 2

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $2n(n + 1)(2n + 1)/6 - 5n(n + 1)/2$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(4n^2 - 9n - 13)/6$ (cso) **A1**

(a) Answer: $n(4n^2 - 9n - 13)/6$

(b) S(8) - S(4) **M1**

(b) S(8) = 228 **A1**

(b) S(4) = 10 **A1**

(b) 218 **A1**

(b) Answer: 218

Question 3

(a) $6[n(n + 1)/2]^2 - 3n(n + 1)(2n + 1)/6$ **M1**

(a) takes out the common factor $n(n + 1)/12$ **M1**

(a) $n(n + 1)(18n^2 + 6n - 6)/12$ (cso) **A1**

(a) Answer: $n(n + 1)(18n^2 + 6n - 6)/12$

(b) S(11) - S(3) **M1**

(b) 24618 - 174 **A1**

(b) 24444 **A1**

(b) Answer: 24444

Question 4

(a) writes $1/r - 1/(r + 4)$ over a common denominator **M1**

(a) = $4/(r(r + 4))$ and multiplies by 1/2 (cso) **A1**

(a) Answer: $2/(r(r + 4)) = (1/2)(1/r - 1/(r + 4))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2 + 1/3 + 1/4) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3) + 1/(n + 4))$ **A1**

(b) $S_n = (1/2)[25/12 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3) - 1/(n + 4)]$ **A1**

(b) $S_{17} = 11237/11970$ **A1**

(b) Answer: $S_{17} = 11237/11970$

(c) the terms in n tend to 0 **M1**

(c) 25/24 **A1**

(c) Answer: 25/24

Question 5

(a) $6[n(n + 1)/2]^2 + 3n(n + 1)(2n + 1)/6$ **M1**

(a) takes out the common factor $n(n + 1)/12$ **M1**

(a) $n(n + 1)(18n^2 + 30n + 6)/12$ (cso) **A1**

(a) Answer: $n(n + 1)(18n^2 + 30n + 6)/12$

(b) S(11) - S(4) **M1**

(b) 27654 - 690 **A1**

(b) 26964 **A1**

(b) Answer: 26964

Question 6

(a) recognising $\sum_{r=n+1}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$ **M1**

(a) substituting the standard result with 2n: $(2n)(2n+1)(4n+1)/6$ **M1**

(a) substituting the standard result with n: $n(n+1)(2n+1)/6$ **M1**

(a) combining over a common denominator and factorising out $n(2n+1)$ **M1**

(a) cso: simplifies to $n(2n+1)(7n+1)/6$ **A1**

(a) Answer: $n(2n+1)(7n+1)/6$

(b) substituting n = 5 into $n(2n+1)(7n+1)/6$ **M1**

(b) 330 cao **A1**

(b) Answer: 330

Question 7

(a) expanding $(r-1)^3 = r^3 - 3r^2 + 3r - 1$ **M1**

(a) cso: $r^3 - (r-1)^3 = 3r^2 - 3r + 1$ **A1**

(a) Answer: $3r^2 - 3r + 1$

(b) recognising the left-hand side telescopes: $\sum_{r=1}^{n} [r^3-(r-1)^3] = n^3 - 0^3 = n^3$ **M1**

(b) writing the right-hand side as 3*sum r^2 - 3*sum r + n and substituting sum r = n(n+1)/2 **M1**

(b) rearranging to make sum r^2 the subject: 3*sum $r^2 = n^3 - n + 3n(n+1)/2$ **M1**

(b) combining over a common denominator and factorising: 3*sum $r^2 = n(n+1)(2n+1)/2$ **M1**

(b) cso: dividing by 3 to obtain sum $r^2 = n(n+1)(2n+1)/6$ **A1**

(b) Answer: $n(n+1)(2n+1)/6$

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