1.10 Series and Summation: 2-hour Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question is about the series with r-th term $u_r = 6/(r(r + 1))$.

(a) Show that $6/(r(r + 1)) = (6)(1/r - 1/(r + 1))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 18. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 1 is 8 marks)

2 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(2r^2 - r + 5)$ can be written as $n(4n^2 + 3n + 29)/6$. **(4)**

(b) Hence evaluate the sum from r = 5 to r = 9 of $(2r^2 - r + 5)$. **(4)**

(Total for Question 2 is 8 marks)

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3 This question is about the series with r-th term $u_r = 4/(r(r + 2))$.

(a) Show that $4/(r(r + 2)) = (2)(1/r - 1/(r + 2))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 19. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 3 is 8 marks)

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4 This question is about the series with r-th term $u_r = 5/(r(r + 4))$.

(a) Show that $5/(r(r + 4)) = (5/4)(1/r - 1/(r + 4))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 9. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 4 is 8 marks)

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5 This question is about the series with r-th term $u_r = 3/(r(r + 4))$.

(a) Show that $3/(r(r + 4)) = (3/4)(1/r - 1/(r + 4))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 16. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 5 is 8 marks)

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6 This question is about the series with r-th term $u_r = 6/(r(r + 2))$.

(a) Show that $6/(r(r + 2)) = (3)(1/r - 1/(r + 2))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 14. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 6 is 8 marks)

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7 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(2r^2 + 4r - 1)$ can be written as $n(4n^2 + 18n + 8)/6$. **(4)**

(b) Hence evaluate the sum from r = 4 to r = 10 of $(2r^2 + 4r - 1)$. **(4)**

(Total for Question 7 is 8 marks)

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8 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(5r^2 + 6r - 1)$ can be written as $n(10n^2 + 33n + 17)/6$. **(4)**

(b) Hence evaluate the sum from r = 3 to r = 17 of $(5r^2 + 6r - 1)$. **(4)**

(Total for Question 8 is 8 marks)

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9 This question is about the series with r-th term $u_r = 1/(r(r + 2))$.

(a) Show that $1/(r(r + 2)) = (1/2)(1/r - 1/(r + 2))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 9. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 9 is 8 marks)

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10 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(3r^2 - 4r + 5)$ can be written as $n(6n^2 - 3n + 21)/6$. **(4)**

(b) Hence evaluate the sum from r = 3 to r = 12 of $(3r^2 - 4r + 5)$. **(4)**

(Total for Question 10 is 8 marks)

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11 This question uses the method of differences on the series with general term $1/((2r-1)(2r+1))$.

(a) Express $1/((2r-1)(2r+1))$ in the form $A/(2r-1) + B/(2r+1)$, where A and B are constants to be found. **(4)**

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} 1/((2r-1)(2r+1)) = n/(2n+1)$. **(4)**

(c) Hence find $\sum_{r=1}^{50} 1/((2r-1)(2r+1))$ as an exact fraction. **(2)**

(d) Find the smallest positive integer n for which $\sum_{r=1}^{n} 1/((2r-1)(2r+1)) > 0.499$. **(3)**

(Total for Question 11 is 13 marks)

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12 This question extends the method of differences to a series with a cubic general term, and then to a quartic general term.

(a) Show that $r(r+1)(r+2)(r+3) - (r-1)r(r+1)(r+2) = 4r(r+1)(r+2)$. **(2)**

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} r(r+1)(r+2) = n(n+1)(n+2)(n+3)/4$. **(4)**

(c) By defining $g(r) = r(r+1)(r+2)(r+3)(r+4)$ and considering $g(r) - g(r-1)$, show that $\sum_{r=1}^{n} r(r+1)(r+2)(r+3) = n(n+1)(n+2)(n+3)(n+4)/5$. **(4)**

(d) Use the result in part (c) to evaluate $\sum_{r=1}^{10} r(r+1)(r+2)(r+3)$. **(2)**

(Total for Question 12 is 12 marks)

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Mark scheme · 1.10 Series and Summation: 2-hour Pack

Question 1

(a) writes 1/r - 1/(r + 1) over a common denominator **M1**

(a) = 1/(r(r + 1)) and multiplies by 6 (cso) **A1**

(a) Answer: 6/(r(r + 1)) = (6)(1/r - 1/(r + 1))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1) - (1/(n + 1)) **A1**

(b) $S_n = (6)[1 - 1/(n + 1)]$ **A1**

(b) $S_{18} = 108/19$ **A1**

(b) Answer: $S_{18} = 108/19$

(c) the terms in n tend to 0 **M1**

(c) 6 **A1**

(c) Answer: 6

Question 2

(a) uses n(n + 1)(2n + 1)/6 for the r^2 term and n(n + 1)/2 for the r term **M1**

(a) writes 2n(n + 1)(2n + 1)/6 - n(n + 1)/2 + 5n over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(4n^2 + 3n + 29)/6$ (cso) **A1**

(a) Answer: $n(4n^2 + 3n + 29)/6$

(b) S(9) - S(4) **M1**

(b) S(9) = 570 **A1**

(b) S(4) = 70 **A1**

(b) 500 **A1**

(b) Answer: 500

Question 3

(a) writes 1/r - 1/(r + 2) over a common denominator **M1**

(a) = 2/(r(r + 2)) and multiplies by 2 (cso) **A1**

(a) Answer: 4/(r(r + 2)) = (2)(1/r - 1/(r + 2))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2) - (1/(n + 1) + 1/(n + 2)) **A1**

(b) $S_n = (2)[3/2 - 1/(n + 1) - 1/(n + 2)]$ **A1**

(b) $S_{19} = 589/210$ **A1**

(b) Answer: $S_{19} = 589/210$

(c) the terms in n tend to 0 **M1**

(c) 3 **A1**

(c) Answer: 3

Question 4

(a) writes 1/r - 1/(r + 4) over a common denominator **M1**

(a) = 4/(r(r + 4)) and multiplies by 5/4 (cso) **A1**

(a) Answer: 5/(r(r + 4)) = (5/4)(1/r - 1/(r + 4))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3 + 1/4) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3) + 1/(n + 4)) **A1**

(b) $S_n = (5/4)[25/12 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3) - 1/(n + 4)]$ **A1**

(b) $S_9 = 2477/1144$ **A1**

(b) Answer: $S_9 = 2477/1144$

(c) the terms in n tend to 0 **M1**

(c) 125/48 **A1**

(c) Answer: 125/48

Question 5

(a) writes 1/r - 1/(r + 4) over a common denominator **M1**

(a) = 4/(r(r + 4)) and multiplies by 3/4 (cso) **A1**

(a) Answer: 3/(r(r + 4)) = (3/4)(1/r - 1/(r + 4))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3 + 1/4) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3) + 1/(n + 4)) **A1**

(b) $S_n = (3/4)[25/12 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3) - 1/(n + 4)]$ **A1**

(b) $S_{16} = 27127/19380$ **A1**

(b) Answer: $S_{16} = 27127/19380$

(c) the terms in n tend to 0 **M1**

(c) 25/16 **A1**

(c) Answer: 25/16

Question 6

(a) writes 1/r - 1/(r + 2) over a common denominator **M1**

(a) = 2/(r(r + 2)) and multiplies by 3 (cso) **A1**

(a) Answer: 6/(r(r + 2)) = (3)(1/r - 1/(r + 2))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2) - (1/(n + 1) + 1/(n + 2)) **A1**

(b) $S_n = (3)[3/2 - 1/(n + 1) - 1/(n + 2)]$ **A1**

(b) $S_{14} = 329/80$ **A1**

(b) Answer: $S_{14} = 329/80$

(c) the terms in n tend to 0 **M1**

(c) 9/2 **A1**

(c) Answer: 9/2

Question 7

(a) uses n(n + 1)(2n + 1)/6 for the r^2 term and n(n + 1)/2 for the r term **M1**

(a) writes 2n(n + 1)(2n + 1)/6 + 4n(n + 1)/2 - n over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(4n^2 + 18n + 8)/6$ (cso) **A1**

(a) Answer: $n(4n^2 + 18n + 8)/6$

(b) S(10) - S(3) **M1**

(b) S(10) = 980 **A1**

(b) S(3) = 49 **A1**

(b) 931 **A1**

(b) Answer: 931

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Question 8

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $5n(n + 1)(2n + 1)/6 + 6n(n + 1)/2 - n$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(10n^2 + 33n + 17)/6$ (cso) **A1**

(a) Answer: $n(10n^2 + 33n + 17)/6$

(b) S(17) - S(2) **M1**

(b) S(17) = 9826 **A1**

(b) S(2) = 41 **A1**

(b) 9785 **A1**

(b) Answer: 9785

Question 9

(a) writes $1/r - 1/(r + 2)$ over a common denominator **M1**

(a) $= 2/(r(r + 2))$ and multiplies by 1/2 (cso) **A1**

(a) Answer: $1/(r(r + 2)) = (1/2)(1/r - 1/(r + 2))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2) - (1/(n + 1) + 1/(n + 2))$ **A1**

(b) $S_n = (1/2)[3/2 - 1/(n + 1) - 1/(n + 2)]$ **A1**

(b) $S_9 = 36/55$ **A1**

(b) Answer: $S_9 = 36/55$

(c) the terms in n tend to 0 **M1**

(c) 3/4 **A1**

(c) Answer: 3/4

Question 10

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $3n(n + 1)(2n + 1)/6 - 4n(n + 1)/2 + 5n$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(6n^2 - 3n + 21)/6$ (cso) **A1**

(a) Answer: $n(6n^2 - 3n + 21)/6$

(b) S(12) - S(2) **M1**

(b) S(12) = 1698 **A1**

(b) S(2) = 13 **A1**

(b) 1685 **A1**

(b) Answer: 1685

Question 11

(a) forming $1 = A(2r+1) + B(2r-1)$ **M1**

(a) substituting r = 1/2 and r = -1/2 (or comparing coefficients) **M1**

(a) A = 1/2 cao **A1**

(a) B = -1/2 cao **A1**

(a) Answer: A = 1/2, B = -1/2

(b) writing sum = (1/2) sum [1/(2r-1) - 1/(2r+1)] **M1**

(b) listing leading/trailing terms to show telescoping: (1/2) [(1/1 - 1/3) + (1/3 - 1/5) + ... + (1/(2n-1) - 1/(2n+1))] **M1**

(b) cancelling middle terms to leave $(1/2)[1 - 1/(2n+1)]$ **M1**

(b) cso: simplifies to $n/(2n+1)$ **A1**

(b) Answer: $n/(2n+1)$

(c) substituting n = 50 into $n/(2n+1)$ **M1**

(c) 50/101 cao **A1**

(c) Answer: 50/101

(d) forming the inequality $n/(2n+1) > 0.499$ **M1**

(d) solving to obtain n > 249.5 (or equivalent correct rearrangement) **M1**

(d) n = 250 cao, with a check that n=249 fails and n=250 succeeds **A1**

(d) Answer: n = 250

Question 12

(a) factorising out the common factor $r(r+1)(r+2)$ **M1**

(a) cso: $r(r+1)(r+2)[(r+3)-(r-1)] = 4r(r+1)(r+2)$ **A1**

(a) Answer: $4r(r+1)(r+2)$

(b) writing $\sum_{r=1}^{n} 4r(r+1)(r+2)$ as $\sum_{r=1}^{n} [f(r)-f(r-1)]$ where $f(r)=r(r+1)(r+2)(r+3)$ **M1**

(b) recognising the sum telescopes to f(n) - f(0) **M1**

(b) using f(0) = 0*1*2*3 = 0 so the sum equals $n(n+1)(n+2)(n+3)$ **M1**

(b) cso: dividing by 4 to obtain sum $r(r+1)(r+2) = n(n+1)(n+2)(n+3)/4$ **A1**

(b) Answer: $n(n+1)(n+2)(n+3)/4$

(c) showing $g(r) - g(r-1) = r(r+1)(r+2)(r+3)[(r+4)-(r-1)] = 5r(r+1)(r+2)(r+3)$ **M1**

(c) summing both sides from r=1 to n so the left-hand side telescopes to g(n) - g(0) **M1**

(c) using g(0) = 0*1*2*3*4 = 0, so sum $5r(r+1)(r+2)(r+3) = n(n+1)(n+2)(n+3)(n+4)$ **A1**

(c) cso: dividing by 5 to obtain sum $r(r+1)(r+2)(r+3) = n(n+1)(n+2)(n+3)(n+4)/5$ **A1**

(c) Answer: $n(n+1)(n+2)(n+3)(n+4)/5$

(d) substituting n = 10 into $n(n+1)(n+2)(n+3)(n+4)/5$ **M1**

(d) 48048 cao **A1**

(d) Answer: 48048

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