1.10 Series and Summation: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The standard results for sums of powers of the first n positive integers are used throughout this worksheet.

(a) State the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$. **(3)**

(b) Calculate $\sum_{r=1}^{20} r$. **(2)**

(c) Calculate $\sum_{r=1}^{12} r^2$. **(2)**

(d) Hence calculate $\sum_{r=1}^{12} (2r^2 - 3r + 1)$. **(3)**

(Total for Question 1 is 10 marks)

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2 A quick check of understanding of σ notation.

(a) Which of the following is equal to $\sum_{r=1}^{n} (2r - 1)$, for all positive integers n? **(1)**

- ☐ A) n^2
- ☐ B) n(n+1)
- ☐ C) n(2n+1)
- ☐ D) $[n(n+1)/2]^2$

(b) Justify your answer to part (a) by expressing $\sum_{r=1}^{n} (2r - 1)$ in terms of n, showing your full working. **(3)**

(Total for Question 2 is 4 marks)

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3 Tom and Freya are comparing partial sums of the series with general term $r^2 - 2r$.

(a) Show, using standard results, that $\sum_{r=5}^{15} (r^2 - 2r) = 990$. **(4)**

(b) A second sum is defined by $\sum_{r=k}^{15} (r^2 - 2r) = 916$, where k is a positive integer with $1 \leq k \leq 15$. Find the value of k. **(4)**

(Total for Question 3 is 8 marks)

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4 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(r^2 + 5r + 4)$ can be written as $n(2n^2 + 18n + 40)/6$. **(4)**

(b) Hence evaluate the sum from r = 6 to r = 12 of $(r^2 + 5r + 4)$. **(4)**

(Total for Question 4 is 8 marks)

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5 This question uses the method of differences on the series with general term $1/(r(r+1))$.

(a) Show that $1/r - 1/(r+1) = 1/(r(r+1))$. **(2)**

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} 1/(r(r+1)) = n/(n+1)$. **(3)**

(c) Hence evaluate $\sum_{r=1}^{99} 1/(r(r+1))$ as an exact fraction. **(2)**

(d) State the value that $\sum_{r=1}^{n} 1/(r(r+1))$ approaches as n tends to infinity, giving a reason. **(2)**

(Total for Question 5 is 9 marks)

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6 Prove, by mathematical induction, that for all positive integers n:

$$\sum_{r=1}^{n} r(2r - 1) = n(n+1)(4n-1)/6$$

(a) Prove the given result by mathematical induction, showing the basis case, the inductive step and a full conclusion. **(7)**

(Total for Question 6 is 7 marks)

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Mark scheme · 1.10 Series and Summation: Test Pack

Question 1

(a) sum r = n(n+1)/2 oe **B1**

(a) sum r^2 = n(n+1)(2n+1)/6 oe **B1**

(a) sum r^3 = $[n(n+1)/2]^2$ oe **B1**

(a) Answer: sum r = n(n+1)/2; sum r^2 = n(n+1)(2n+1)/6; sum r^3 = $[n(n+1)/2]^2$

(b) substituting n = 20 into n(n+1)/2 **M1**

(b) 210 cao **A1**

(b) Answer: 210

(c) substituting n = 12 into n(n+1)(2n+1)/6 **M1**

(c) 650 cao **A1**

(c) Answer: 650

(d) splitting into 2*sum r^2 - 3*sum r + sum 1 **M1**

(d) substituting sum r^2 = 650 (or recomputed) and sum r = 78 **M1**

(d) 1078 cao **A1**

(d) Answer: 1078

Question 2

(a) A cao **B1**

(a) Answer: A) n^2

(b) splitting into 2*sum r - sum 1 **M1**

(b) substituting sum r = n(n+1)/2 to get n(n+1) - n **M1**

(b) cso: simplifies to n^2 **A1**

(b) Answer: n^2

Question 3

(a) splitting into $\sum_{r=5}^{15} r^2 - 2*\sum_{r=5}^{15} r$ **M1**

(a) using $\sum_{r=5}^{15} r^2 = \sum_{r=1}^{15} r^2 - \sum_{r=1}^{4} r^2 = 1240 - 30 = 1210$ **M1**

(a) using $\sum_{r=5}^{15} r = \sum_{r=1}^{15} r - \sum_{r=1}^{4} r = 120 - 10 = 110$ **M1**

(a) cso: 1210 - 2(110) = 990 **A1**

(a) Answer: 990

(b) expressing $\sum_{r=k}^{15} (r^2-2r)$ as $\sum_{r=1}^{15} (r^2-2r) - \sum_{r=1}^{k-1} (r^2-2r)$ **M1**

(b) computing $\sum_{r=1}^{15} (r^2-2r) = 1240 - 2(120) = 1000$ **M1**

(b) forming and solving $1000 - \sum_{r=1}^{k-1} (r^2-2r) = 916$, i.e. $\sum_{r=1}^{k-1} (r^2-2r) = 84$, by trial or algebraically **M1**

(b) k = 8 cao **A1**

(b) Answer: k = 8

Question 4

(a) uses n(n + 1)(2n + 1)/6 for the r^2 term and n(n + 1)/2 for the r term **M1**

(a) writes n(n + 1)(2n + 1)/6 + 5n(n + 1)/2 + 4n over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(2n^2 + 18n + 40)/6$ (cso) **A1**

(a) Answer: $n(2n^2 + 18n + 40)/6$

(b) S(12) - S(5) **M1**

(b) S(12) = 1088 **A1**

(b) S(5) = 150 **A1**

(b) 938 **A1**

(b) Answer: 938

Question 5

(a) combining 1/r - 1/(r+1) over the common denominator r(r+1) **M1**

(a) cso: numerator (r+1) - r = 1, giving 1/(r(r+1)) **A1**

(a) Answer: 1/(r(r+1))

(b) writing out enough leading and trailing terms to show the telescoping pattern, e.g. (1 - 1/2) + (1/2 - 1/3) + ... + (1/n - 1/(n+1)) **M1**

(b) cancelling all middle terms to leave 1 - 1/(n+1) **M1**

(b) cso: simplifies to n/(n+1) **A1**

(b) Answer: n/(n+1)

(c) substituting n = 99 into n/(n+1) **M1**

(c) 99/100 cao **A1**

(c) Answer: 99/100

(d) 1 cao **B1**

(d) correct reason, e.g. n/(n+1) = 1/(1 + 1/n) -> 1 as n -> infinity, since 1/n -> 0 **B1**

(d) Answer: 1

Question 6

(a) basis: n=1 gives LHS = 1*1 = 1 and RHS = 1*2*3/6 = 1, so true for n=1 **B1**

(a) assuming true for n=k and adding the (k+1)th term (k+1)(2(k+1)-1) = (k+1)(2k+1) to both sides **M1**

(a) combining over a common denominator of 6: (k+1)[k(4k-1) + 6(2k+1)]/6 **M1**

(a) correctly simplifying the bracket to $4k^2 + 11k + 6$ oe **A1**

(a) factorising $4k^2+11k+6$ = (k+2)(4k+3) **M1**

(a) cso: obtaining (k+1)(k+2)(4k+3)/6, which is the given formula with n = k+1 **A1**

(a) dependent on all previous marks: correct conclusion stating true for n=1, and if true for n=k then true for n=k+1, so by induction true for all positive integers n **dB1**

(a) Answer: Proved by induction: true for n=1; if true for n=k then true for n=k+1; therefore true for all positive integers n.

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