1.10 Series and Summation

EDEXCEL 9FM0 · Calculator allowed · about 235 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Priya is investigating the series with general term $u_r = 4r + 3$, for r = 1, 2, 3, ...

(a) Show that $\sum_{r=1}^{n} (4r + 3) = n(2n + 5)$. **(3)**

(b) Hence find the value of n for which $\sum_{r=1}^{n} (4r + 3) = 900$. **(3)**

(c) State why the other root of the quadratic in part (b) is rejected. **(1)**

(Total for Question 1 is 7 marks)

2 Aisha and Daniel are exploring sums of the positive integers and their subsets.

(a) Find the smallest positive integer n such that $\sum_{r=1}^{n} r > 1000$. **(3)**

(b) Calculate $\sum_{r=1}^{40} 2r$, the sum of the first 40 positive even integers. **(2)**

(c) Hence, or otherwise, find the sum of the first 40 positive odd integers. **(2)**

(Total for Question 2 is 7 marks)

3 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(3r^2 - 5r + 5)$ can be written as $n(6n^2 - 6n + 18)/6$. **(4)**

(b) Hence evaluate the sum from r = 4 to r = 17 of $(3r^2 - 5r + 5)$. **(4)**

(Total for Question 3 is 8 marks)

4 Use the method of differences, or standard results, to answer this question.

(a) Show that the sum from r = 1 to n of $5r(r + 1)(r + 2) = 5n(n + 1)(n + 2)(n + 3)/4$. **(3)**

(b) Hence evaluate the sum from r = 4 to 18 of $5r(r + 1)(r + 2)$. **(2)**

(Total for Question 4 is 5 marks)

5 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(2r^2 - 5r + 0)$ can be written as $n(4n^2 - 9n - 13)/6$. **(4)**

(b) Hence evaluate the sum from r = 5 to r = 8 of $(2r^2 - 5r + 0)$. **(4)**

(Total for Question 5 is 8 marks)

6 Use the standard results for the sums of r^2 and r^3 to answer this question.

(a) Show that the sum from r = 1 to n of $(6r^3 - 3r^2)$ is $n(n + 1)(18n^2 + 6n - 6)/12$. **(3)**

(b) Hence find the value of the sum from r = 4 to r = 11 of $(6r^3 - 3r^2)$. **(3)**

(Total for Question 6 is 6 marks)

7 This question is about the series with r-th term $u_r = 2/(r(r + 4))$.

(a) Show that $2/(r(r + 4)) = (1/2)(1/r - 1/(r + 4))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 17. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 7 is 8 marks)

8 Use the standard results for the sums of r^2 and r^3 to answer this question.

(a) Show that the sum from r = 1 to n of $(6r^3 + 3r^2)$ is $n(n + 1)(18n^2 + 30n + 6)/12$. **(3)**

(b) Hence find the value of the sum from r = 5 to r = 11 of $(6r^3 + 3r^2)$. **(3)**

(Total for Question 8 is 6 marks)

9 This question is about the series with r-th term $u_r = 6/(r(r + 1))$.

(a) Show that $6/(r(r + 1)) = (6)(1/r - 1/(r + 1))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 18. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 9 is 8 marks)

10 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(2r^2 - r + 5)$ can be written as $n(4n^2 + 3n + 29)/6$. **(4)**

(b) Hence evaluate the sum from r = 5 to r = 9 of $(2r^2 - r + 5)$. **(4)**

(Total for Question 10 is 8 marks)

11 This question is about the series with r-th term $u_r = 4/(r(r + 2))$.

(a) Show that $4/(r(r + 2)) = (2)(1/r - 1/(r + 2))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 19. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 11 is 8 marks)

12 This question is about the series with r-th term $u_r = 5/(r(r + 4))$.

(a) Show that $5/(r(r + 4)) = (5/4)(1/r - 1/(r + 4))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 9. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 12 is 8 marks)

13 This question is about the series with r-th term $u_r = 3/(r(r + 4))$.

(a) Show that $3/(r(r + 4)) = (3/4)(1/r - 1/(r + 4))$. **(2)**

(b) Use the method of differences to find the sum from $r = 1$ to n of u_r, and evaluate the sum from $r = 1$ to 16. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 13 is 8 marks)

14 This question is about the series with r-th term $u_r = 6/(r(r + 2))$.

(a) Show that $6/(r(r + 2)) = (3)(1/r - 1/(r + 2))$. **(2)**

(b) Use the method of differences to find the sum from $r = 1$ to n of u_r, and evaluate the sum from $r = 1$ to 14. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 14 is 8 marks)

15 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(2r^2 + 4r - 1)$ can be written as $n(4n^2 + 18n + 8)/6$. **(4)**

(b) Hence evaluate the sum from r = 4 to r = 10 of $(2r^2 + 4r - 1)$. **(4)**

(Total for Question 15 is 8 marks)

16 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(5r^2 + 6r - 1)$ can be written as $n(10n^2 + 33n + 17)/6$. **(4)**

(b) Hence evaluate the sum from r = 3 to r = 17 of $(5r^2 + 6r - 1)$. **(4)**

(Total for Question 16 is 8 marks)

17 This question is about the series with r-th term $u_r = 1/(r(r + 2))$.

(a) Show that $1/(r(r + 2)) = (1/2)(1/r - 1/(r + 2))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 9. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 17 is 8 marks)

18 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(3r^2 - 4r + 5)$ can be written as $n(6n^2 - 3n + 21)/6$. **(4)**

(b) Hence evaluate the sum from r = 3 to r = 12 of $(3r^2 - 4r + 5)$. **(4)**

(Total for Question 18 is 8 marks)

19 Consider the series with general term r(r+1), for r = 1, 2, 3, ...

(a) Show that $\sum_{r=1}^{n} r(r+1) = n(n+1)(n+2)/3$. **(4)**

(b) Hence evaluate $\sum_{r=1}^{25} r(r+1)$. **(2)**

(c) Use your answer to part (a) to find $\sum_{r=10}^{25} r(r+1)$. **(3)**

(Total for Question 19 is 9 marks)

20 For a positive integer n, consider $\sum_{r=n+1}^{2n} r^2$, the sum of the squares of the integers from n+1 to 2n.

(a) Show that $\sum_{r=n+1}^{2n} r^2 = n(2n+1)(7n+1)/6$. **(5)**

(b) Hence find $\sum_{r=6}^{10} r^2$ by using the result in part (a) with n = 5. **(2)**

(Total for Question 20 is 7 marks)

21 This question uses the method of differences on the series with general term $1/((2r-1)(2r+1))$.

(a) Express $1/((2r-1)(2r+1))$ in the form $A/(2r-1) + B/(2r+1)$, where A and B are constants to be found. **(4)**

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} 1/((2r-1)(2r+1)) = n/(2n+1)$. **(4)**

(c) Hence find $\sum_{r=1}^{50} 1/((2r-1)(2r+1))$ as an exact fraction. **(2)**

(d) Find the smallest positive integer n for which $\sum_{r=1}^{n} 1/((2r-1)(2r+1)) > 0.499$. **(3)**

(Total for Question 21 is 13 marks)

22 This question derives the standard result for $\sum_{r=1}^{n} r^2$ from first principles, using the method of differences.

(a) Show that $r^3 - (r-1)^3 = 3r^2 - 3r + 1$. **(2)**

(b) By summing both sides of the identity in part (a) from r = 1 to n, and using the method of differences on the left-hand side, show that $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$. **(5)**

(You may use $\sum_{r=1}^{n} r = n(n+1)/2$ without proof.)

(Total for Question 22 is 7 marks)

23 This question extends the method of differences to a series with a cubic general term, and then to a quartic general term.

(a) Show that $r(r+1)(r+2)(r+3) - (r-1)r(r+1)(r+2) = 4r(r+1)(r+2)$. **(2)**

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} r(r+1)(r+2) = n(n+1)(n+2)(n+3)/4$. **(4)**

(c) By defining $g(r) = r(r+1)(r+2)(r+3)(r+4)$ and considering $g(r) - g(r-1)$, show that $\sum_{r=1}^{n} r(r+1)(r+2)(r+3) = n(n+1)(n+2)(n+3)(n+4)/5$. **(4)**

(d) Use the result in part (c) to evaluate $\sum_{r=1}^{10} r(r+1)(r+2)(r+3)$. **(2)**

(Total for Question 23 is 12 marks)

Mark scheme · 1.10 Series and Summation

Question 1

(a) splitting sum into 4*sum r + sum 3 **M1**

(a) substituting sum r = n(n+1)/2 to get 2n(n+1) + 3n **M1**

(a) cso: simplifies to n(2n+5) **A1**

(a) Answer: n(2n + 5)

(b) forming $2n^2 + 5n - 900 = 0$ **M1**

(b) solving the quadratic (formula or factorisation) to get n = 20 or n = -22.5 **M1**

(b) n = 20 cao, rejecting the non-integer negative root **A1**

(b) Answer: n = 20

(c) n must be a positive integer (number of terms), so the negative/non-integer root has no meaning here **B1**

(c) Answer: n represents a number of terms, so it must be a positive integer; the other root is negative.

Question 2

(a) forming the inequality n(n+1)/2 > 1000, i.e. n(n+1) > 2000 **M1**

(a) trial (or solving the quadratic) to compare n=44 and n=45 **M1**

(a) n = 45 cao, with 44*45=1980<2000 and 45*46=2070>2000 shown **A1**

(a) Answer: n = 45

(b) using sum 2r = 2*sum r with n=40 **M1**

(b) 1640 cao **A1**

(b) Answer: 1640

(c) valid method, e.g. $\sum_{r=1}^{80} r - \sum_{r=1}^{40} 2r$, or using $\text{sum}(2r-1)=n^2$ with n=40 **M1**

(c) 1600 cao **A1**

(c) Answer: 1600

Question 3

(a) uses n(n + 1)(2n + 1)/6 for the r^2 term and n(n + 1)/2 for the r term **M1**

(a) writes 3n(n + 1)(2n + 1)/6 - 5n(n + 1)/2 + 5n over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(6n^2 - 6n + 18)/6$ (cso) **A1**

(a) Answer: $n(6n^2 - 6n + 18)/6$

(b) S(17) - S(3) **M1**

(b) S(17) = 4675 **A1**

(b) S(3) = 27 **A1**

(b) 4648 **A1**

(b) Answer: 4648

Question 4

(a) uses the identity r(r + 1)(r + 2) = [r(r + 1)(r + 2)(r + 3) - (r - 1)r(r + 1)(r + 2)]/4 **M1**

(a) applies the method of differences so that intermediate terms cancel **M1**

(a) obtains n(n + 1)(n + 2)(n + 3)/4 (cso) **A1**

(a) Answer: 5n(n + 1)(n + 2)(n + 3)/4

(b) S(18) - S(3) **M1**

(b) 179100 **A1**

(b) Answer: 179100

Question 5

(a) uses n(n + 1)(2n + 1)/6 for the r^2 term and n(n + 1)/2 for the r term **M1**

(a) writes 2n(n + 1)(2n + 1)/6 - 5n(n + 1)/2 over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(4n^2 - 9n - 13)/6$ (cso) **A1**

(a) Answer: $n(4n^2 - 9n - 13)/6$

(b) S(8) - S(4) **M1**

(b) S(8) = 228 **A1**

(b) S(4) = 10 **A1**

(b) 218 **A1**

(b) Answer: 218

Question 6

(a) $6[n(n + 1)/2]^2 - 3n(n + 1)(2n + 1)/6$ **M1**

(a) takes out the common factor n(n + 1)/12 **M1**

(a) $n(n + 1)(18n^2 + 6n - 6)/12$ (cso) **A1**

(a) Answer: $n(n + 1)(18n^2 + 6n - 6)/12$

(b) S(11) - S(3) **M1**

(b) 24618 - 174 **A1**

(b) 24444 **A1**

(b) Answer: 24444

Question 7

(a) writes 1/r - 1/(r + 4) over a common denominator **M1**

(a) = 4/(r(r + 4)) and multiplies by 1/2 (cso) **A1**

(a) Answer: 2/(r(r + 4)) = (1/2)(1/r - 1/(r + 4))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3 + 1/4) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3) + 1/(n + 4)) **A1**

(b) $S_n = (1/2)[25/12 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3) - 1/(n + 4)]$ **A1**

(b) $S_{17} = 11237/11970$ **A1**

(b) Answer: $S_{17} = 11237/11970$

(c) the terms in n tend to 0 **M1**

(c) 25/24 **A1**

(c) Answer: 25/24

Question 8

(a) $6[n(n + 1)/2]^2 + 3n(n + 1)(2n + 1)/6$ **M1**

(a) takes out the common factor $n(n + 1)/12$ **M1**

(a) $n(n + 1)(18n^2 + 30n + 6)/12$ (cso) **A1**

(a) Answer: $n(n + 1)(18n^2 + 30n + 6)/12$

(b) $S(11) - S(4)$ **M1**

(b) 27654 - 690 **A1**

(b) 26964 **A1**

(b) Answer: 26964

Question 9

(a) writes $1/r - 1/(r + 1)$ over a common denominator **M1**

(a) $= 1/(r(r + 1))$ and multiplies by 6 (cso) **A1**

(a) Answer: $6/(r(r + 1)) = (6)(1/r - 1/(r + 1))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1) - (1/(n + 1))$ **A1**

(b) $S_n = (6)[1 - 1/(n + 1)]$ **A1**

(b) $S_{18} = 108/19$ **A1**

(b) Answer: $S_{18} = 108/19$

(c) the terms in n tend to 0 **M1**

(c) 6 **A1**

(c) Answer: 6

Question 10

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $2n(n + 1)(2n + 1)/6 - n(n + 1)/2 + 5n$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(4n^2 + 3n + 29)/6$ (cso) **A1**

(a) Answer: $n(4n^2 + 3n + 29)/6$

(b) $S(9) - S(4)$ **M1**

(b) $S(9) = 570$ **A1**

(b) $S(4) = 70$ **A1**

(b) 500 **A1**

(b) Answer: 500

Question 11

(a) writes $1/r - 1/(r + 2)$ over a common denominator **M1**

(a) $= 2/(r(r + 2))$ and multiplies by 2 (cso) **A1**

(a) Answer: $4/(r(r + 2)) = (2)(1/r - 1/(r + 2))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2) - (1/(n + 1) + 1/(n + 2))$ **A1**

(b) $S_n = (2)[3/2 - 1/(n + 1) - 1/(n + 2)]$ **A1**

(b) $S_{19} = 589/210$ **A1**

(b) Answer: $S_{19} = 589/210$

(c) the terms in n tend to 0 **M1**

(c) 3 **A1**

(c) Answer: 3

Question 12

(a) writes $1/r - 1/(r + 4)$ over a common denominator **M1**

(a) $= 4/(r(r + 4))$ and multiplies by 5/4 (cso) **A1**

(a) Answer: $5/(r(r + 4)) = (5/4)(1/r - 1/(r + 4))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2 + 1/3 + 1/4) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3) + 1/(n + 4))$ **A1**

(b) $S_n = (5/4)[25/12 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3) - 1/(n + 4)]$ **A1**

(b) $S_9 = 2477/1144$ **A1**

(b) Answer: $S_9 = 2477/1144$

(c) the terms in n tend to 0 **M1**

(c) 125/48 **A1**

(c) Answer: 125/48

Question 13

(a) writes $1/r - 1/(r + 4)$ over a common denominator **M1**

(a) $= 4/(r(r + 4))$ and multiplies by 3/4 (cso) **A1**

(a) Answer: $3/(r(r + 4)) = (3/4)(1/r - 1/(r + 4))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2 + 1/3 + 1/4) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3) + 1/(n + 4))$ **A1**

(b) $S_n = (3/4)[25/12 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3) - 1/(n + 4)]$ **A1**

(b) $S_{16} = 27127/19380$ **A1**

(b) Answer: $S_{16} = 27127/19380$

(c) the terms in n tend to 0 **M1**

(c) 25/16 **A1**

(c) Answer: 25/16

Question 14

(a) writes $1/r - 1/(r + 2)$ over a common denominator **M1**

(a) $= 2/(r(r + 2))$ and multiplies by 3 (cso) **A1**

(a) Answer: $6/(r(r + 2)) = (3)(1/r - 1/(r + 2))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2) - (1/(n + 1) + 1/(n + 2))$ **A1**

(b) $S_n = (3)[3/2 - 1/(n + 1) - 1/(n + 2)]$ **A1**

(b) $S_{14} = 329/80$ **A1**

(b) Answer: $S_{14} = 329/80$

(c) the terms in n tend to 0 **M1**

(c) 9/2 **A1**

(c) Answer: 9/2

Question 15

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $2n(n + 1)(2n + 1)/6 + 4n(n + 1)/2 - n$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(4n^2 + 18n + 8)/6$ (cso) **A1**

(a) Answer: $n(4n^2 + 18n + 8)/6$

(b) $S(10) - S(3)$ **M1**

(b) $S(10) = 980$ **A1**

(b) $S(3) = 49$ **A1**

(b) 931 **A1**

(b) Answer: 931

Question 16

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $5n(n + 1)(2n + 1)/6 + 6n(n + 1)/2 - n$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(10n^2 + 33n + 17)/6$ (cso) **A1**

(a) Answer: $n(10n^2 + 33n + 17)/6$

(b) S(17) - S(2) **M1**

(b) S(17) = 9826 **A1**

(b) S(2) = 41 **A1**

(b) 9785 **A1**

(b) Answer: 9785

Question 17

(a) writes $1/r - 1/(r + 2)$ over a common denominator **M1**

(a) $= 2/(r(r + 2))$ and multiplies by 1/2 (cso) **A1**

(a) Answer: $1/(r(r + 2)) = (1/2)(1/r - 1/(r + 2))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2) - (1/(n + 1) + 1/(n + 2))$ **A1**

(b) $S_n = (1/2)[3/2 - 1/(n + 1) - 1/(n + 2)]$ **A1**

(b) $S_9 = 36/55$ **A1**

(b) Answer: $S_9 = 36/55$

(c) the terms in n tend to 0 **M1**

(c) 3/4 **A1**

(c) Answer: 3/4

Question 18

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $3n(n + 1)(2n + 1)/6 - 4n(n + 1)/2 + 5n$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(6n^2 - 3n + 21)/6$ (cso) **A1**

(a) Answer: $n(6n^2 - 3n + 21)/6$

(b) S(12) - S(2) **M1**

(b) S(12) = 1698 **A1**

(b) S(2) = 13 **A1**

(b) 1685 **A1**

(b) Answer: 1685

Question 19

(a) expanding $r(r+1) = r^2 + r$ **M1**

(a) substituting both standard results: sum r^2 + sum r = $n(n+1)(2n+1)/6 + n(n+1)/2$ **M1**

(a) combining over a common denominator of 6 and factorising out n(n+1) **M1**

(a) cso: simplifies to $n(n+1)(n+2)/3$ **A1**

(a) Answer: $n(n+1)(n+2)/3$

(b) substituting n = 25 into the result of part (a) **M1**

(b) 5850 cao **A1**

(b) Answer: 5850

(c) recognising $\sum_{r=10}^{25} = \sum_{r=1}^{25} - \sum_{r=1}^{9}$ **M1**

(c) computing $\sum_{r=1}^{9} r(r+1) = 9*10*11/3 = 330$ **M1**

(c) 5520 cao **A1**

(c) Answer: 5520

Question 20

(a) recognising $\sum_{r=n+1}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$ **M1**

(a) substituting the standard result with 2n: (2n)(2n+1)(4n+1)/6 **M1**

(a) substituting the standard result with n: n(n+1)(2n+1)/6 **M1**

(a) combining over a common denominator and factorising out n(2n+1) **M1**

(a) cso: simplifies to n(2n+1)(7n+1)/6 **A1**

(a) Answer: n(2n+1)(7n+1)/6

(b) substituting n = 5 into n(2n+1)(7n+1)/6 **M1**

(b) 330 cao **A1**

(b) Answer: 330

Question 21

(a) forming $1 = A(2r+1) + B(2r-1)$ **M1**

(a) substituting r = 1/2 and r = -1/2 (or comparing coefficients) **M1**

(a) A = 1/2 cao **A1**

(a) B = -1/2 cao **A1**

(a) Answer: A = 1/2, B = -1/2

(b) writing sum = (1/2) sum [1/(2r-1) - 1/(2r+1)] **M1**

(b) listing leading/trailing terms to show telescoping: (1/2)[(1/1 - 1/3) + (1/3 - 1/5) + ... + (1/(2n-1) - 1/(2n+1))] **M1**

(b) cancelling middle terms to leave (1/2)[1 - 1/(2n+1)] **M1**

(b) cso: simplifies to n/(2n+1) **A1**

(b) Answer: n/(2n+1)

(c) substituting n = 50 into n/(2n+1) **M1**

(c) 50/101 cao **A1**

(c) Answer: 50/101

(d) forming the inequality n/(2n+1) > 0.499 **M1**

(d) solving to obtain n > 249.5 (or equivalent correct rearrangement) **M1**

(d) n = 250 cao, with a check that n=249 fails and n=250 succeeds **A1**

(d) Answer: n = 250

Question 22

(a) expanding $(r-1)^3 = r^3 - 3r^2 + 3r - 1$ **M1**

(a) cso: $r^3 - (r-1)^3 = 3r^2 - 3r + 1$ **A1**

(a) Answer: $3r^2 - 3r + 1$

(b) recognising the left-hand side telescopes: $\sum_{r=1}^{n} [r^3-(r-1)^3] = n^3 - 0^3 = n^3$ **M1**

(b) writing the right-hand side as 3*sum r^2 - 3*sum r + n and substituting sum r = n(n+1)/2 **M1**

(b) rearranging to make sum r^2 the subject: 3*sum $r^2 = n^3$ - n + 3n(n+1)/2 **M1**

(b) combining over a common denominator and factorising: 3*sum r^2 = n(n+1)(2n+1)/2 **M1**

(b) cso: dividing by 3 to obtain sum r^2 = n(n+1)(2n+1)/6 **A1**

(b) Answer: n(n+1)(2n+1)/6

Question 23

(a) factorising out the common factor r(r+1)(r+2) **M1**

(a) cso: r(r+1)(r+2)[(r+3)-(r-1)] = 4r(r+1)(r+2) **A1**

(a) Answer: 4r(r+1)(r+2)

(b) writing $\sum_{r=1}^{n} 4r(r+1)(r+2)$ as $\sum_{r=1}^{n} [f(r)-f(r-1)]$ where f(r)=r(r+1)(r+2)(r+3) **M1**

(b) recognising the sum telescopes to f(n) - f(0) **M1**

(b) using f(0) = 0*1*2*3 = 0 so the sum equals n(n+1)(n+2)(n+3) **M1**

(b) cso: dividing by 4 to obtain sum r(r+1)(r+2) = n(n+1)(n+2)(n+3)/4 **A1**

(b) Answer: n(n+1)(n+2)(n+3)/4

(c) showing g(r) - g(r-1) = r(r+1)(r+2)(r+3)[(r+4)-(r-1)] = 5r(r+1)(r+2)(r+3) **M1**

(c) summing both sides from r=1 to n so the left-hand side telescopes to g(n) - g(0) **M1**

(c) using g(0) = 0*1*2*3*4 = 0, so sum 5r(r+1)(r+2)(r+3) = n(n+1)(n+2)(n+3)(n+4) **A1**

(c) cso: dividing by 5 to obtain sum r(r+1)(r+2)(r+3) = n(n+1)(n+2)(n+3)(n+4)/5 **A1**

(c) Answer: n(n+1)(n+2)(n+3)(n+4)/5

(d) substituting n = 10 into n(n+1)(n+2)(n+3)(n+4)/5 **M1**

(d) 48048 cao **A1**

(d) Answer: 48048