1.11 Maclaurin Series: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Consider the function $f(x) = e^x \sin x$.

(a) Write down the standard Maclaurin series for e^x and for $\sin x$, in each case up to and including the term in x^4. **(2)**

(b) By multiplying these two series together, find the Maclaurin series for $e^x \sin x$, in ascending powers of x, up to and including the term in x^4, giving each coefficient as an exact fraction (or integer). **(5)**

(Total for Question 1 is 7 marks)

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2 Consider the function $f(x) = e^x \sqrt{1+x}$.

(a) Write down the binomial expansion of $(1+x)^{1/2}$, in ascending powers of x, up to and including the term in x^3, simplifying each coefficient. **(3)**

(b) Using the standard Maclaurin series for e^x, find the Maclaurin series for $f(x) = e^x \sqrt{1+x}$, in ascending powers of x, up to and including the term in x^3, giving each coefficient as a fraction in its simplest form. **(5)**

(Total for Question 2 is 8 marks)

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3 $f(x) = \sqrt{1 + \sin(x)}$.

(a) Find the Maclaurin series for $f(x)$ up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of $f(0)$, $f'(0)$ and $f''(0)$. **(3)**

(Total for Question 3 is 7 marks)

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4 $f(x) = e^{\sin(2x)}$.

(a) Find the Maclaurin series for $f(x)$ up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of $f(0)$, $f'(0)$ and $f''(0)$. **(3)**

(Total for Question 4 is 7 marks)

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5 $f(x) = e^{3x}/(1 + 3x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 5 is 6 marks)

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6 $f(x) = \cos(\sin(2x))$.

(a) Find the Maclaurin series for f(x) up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of f(0), f'(0) and f''(0). **(3)**

(Total for Question 6 is 7 marks)

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7 Let $f(x) = \sec x$.

(a) Show that $f'(x) = \sec x \tan x$, and hence show that $f''(x) = \sec x \tan^2 x + \sec^3 x$. State the value of f''(0). **(5)**

(b) Given further that $f'''(x) = \sec x \tan^3 x + 5 \sec^3 x \tan x$, show that f''''(0) = 5. **(4)**

(c) Hence show that the Maclaurin series for sec x, up to and including the term in x^4, is $1 + x^2/2 + 5x^4/24$. **(2)**

(Total for Question 7 is 11 marks)

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Mark scheme · 1.11 Maclaurin Series: 1-hour Pack

Question 1

(a) $e^x = 1 + x + x^2/2 + x^3/6 + x^4/24$ **B1**

(a) $\sin x = x - x^3/6 (+ 0x^4)$ **B1**

(a) Answer: $e^x = 1+x+x^2/2+x^3/6+x^4/24$; $\sin x = x - x^3/6$

(b) attempts to multiply the two series, collecting terms of equal power up to x^4 **M1**

(b) coefficient of x is 1 **A1**

(b) coefficient of x^2 is 1 **A1**

(b) coefficient of x^3 is 1/3 **A1**

(b) coefficient of x^4 is 0, so $e^x \sin x = x + x^2 + x^3/3 (+ 0x^4)$ cao **A1**

(b) Answer: $e^x \sin x = x + x^2 + x^3/3$

Question 2

(a) uses $(1+x)^n = 1 + nx + n(n-1)x^2/2! + n(n-1)(n-2)x^3/3!$ with n = 1/2 **M1**

(a) coefficient of x^2 simplifies to -1/8 **A1**

(a) coefficient of x^3 simplifies to 1/16, giving $1 + x/2 - x^2/8 + x^3/16$ cao **A1**

(a) Answer: $(1+x)^{1/2} = 1 + x/2 - x^2/8 + x^3/16$

(b) attempts to multiply the series for e^x and for $(1+x)^{1/2}$, collecting terms up to x^3 **M1**

(b) constant term is 1 **A1**

(b) coefficient of x is 3/2 **A1**

(b) coefficient of x^2 is 7/8 **A1**

(b) coefficient of x^3 is 17/48 cao **A1**

(b) Answer: $e^x \sqrt{1+x} = 1 + (3/2)x + (7/8)x^2 + (17/48)x^3$

Question 3

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 + (1/2)x - (1/8)x^2 - (1/48)x^3 + (1/384)x^4$ **A1**

(a) Answer: $1 + (1/2)x - (1/8)x^2 - (1/48)x^3 + (1/384)x^4$

(b) f(0) = 1 **B1**

(b) f'(0) = 1/2 **B1**

(b) f''(0) = 2 x coefficient of x^2 = -1/4 **B1**

(b) Answer: f(0) = 1, f'(0) = 1/2, f''(0) = -1/4

Question 4

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 + 2x + 2x^2 - 2x^4$ **A1**

(a) Answer: $1 + 2x + 2x^2 - 2x^4$

(b) f(0) = 1 **B1**

(b) f'(0) = 2 **B1**

(b) f''(0) = 2 x coefficient of x^2 = 4 **B1**

(b) Answer: f(0) = 1, f'(0) = 2, f''(0) = 4

Question 5

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 + (9/2)x^2 - 9x^3 + (243/8)x^4$ **A1**

(a) Answer: $1 + (9/2)x^2 - 9x^3 + (243/8)x^4$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 1.03904 **A1**

(b) Answer: 1.03904

Question 6

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 - 2x^2 + (10/3)x^4$ **A1**

(a) Answer: $1 - 2x^2 + (10/3)x^4$

(b) f(0) = 1 **B1**

(b) f'(0) = 0 **B1**

(b) f''(0) = 2 x coefficient of x^2 = -4 **B1**

(b) Answer: f(0) = 1, f'(0) = 0, f''(0) = -4

Question 7

(a) differentiates $f(x) = (\cos x)^{-1}$ using the chain rule **M1**

(a) cso: $f'(x) = \sin x (\cos x)^{-2} = \sec x \tan x$ **A1**

(a) differentiates $f'(x) = \sec x \tan x$ using the product rule: $\sec x \tan x \cdot \tan x + \sec x \cdot \sec^2 x$ **M1**

(a) cso: $f''(x) = \sec x \tan^2 x + \sec^3 x$ as required **A1**

(a) f''(0) = 0 + 1 = 1 cao **B1**

(a) Answer: $f''(x) = \sec x \tan^2 x + \sec^3 x$, and $f''(0) = 1$

(b) differentiates $\sec x \tan^3 x$ using the product rule to get $\sec x \tan^4 x + 3\sec^3 x \tan^2 x$ **M1**

(b) differentiates $5\sec^3 x \tan x$ using the product rule to get $15\sec^3 x \tan^2 x + 5\sec^5 x$ **M1**

(b) combines to $f''''(x) = \sec x \tan^4 x + 18\sec^3 x \tan^2 x + 5\sec^5 x$ **A1**

(b) cso: substitutes x=0 (sec0=1, tan0=0) to get f''''(0) = 0 + 0 + 5 = 5 as required **A1**

(b) Answer: f''''(0) = 5

(c) substitutes f(0)=1, f'(0)=0, f''(0)=1, f'''(0)=0, f''''(0)=5 into the Maclaurin formula **M1**

(c) cso: $1 + 0 + x^2/2! + 0 + 5x^4/4! = 1 + x^2/2 + 5x^4/24$ as required **A1**

(c) Answer: $\sec x = 1 + x^2/2 + 5x^4/24$

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