1.11 Maclaurin Series: 30-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 You may use, without proof, the standard Maclaurin series $e^x = 1 + x + x^2/2! + x^3/3! + ...$ and $\sin x = x - x^3/3! + x^5/5! - ...$, both valid for all real x.

(a) By substituting into the series for e^x, find the Maclaurin series for e^{2x}, in ascending powers of x, up to and including the term in x^3. **(3)**

(b) By substituting into the series for sin x, find the Maclaurin series for sin(3x), in ascending powers of x, up to and including the term in x^5, giving each coefficient as an exact fraction. **(4)**

(Total for Question 1 is 7 marks)

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2 Maclaurin series can be used to evaluate limits that are not directly computable by substitution.

(a) By using the Maclaurin series for e^x, find $\lim(x\to 0)$ of $(e^x - 1 - x)/x^2$. **(3)**

(b) By using the Maclaurin series for cos x, find $\lim(x\to 0)$ of $(1 - \cos x - x^2/2)/x^4$. **(4)**

(Total for Question 2 is 7 marks)

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3 Consider the function $h(x) = \sin(x^2)$.

(a) Using the standard Maclaurin series for sin x, show that the Maclaurin series for $\sin(x^2)$, up to and including the term in x^{10}, is $x^2 - x^6/6 + x^{10}/120$. **(3)**

(b) By integrating this series term by term, find an estimate for the integral from 0 to 0.5 of $\sin(x^2)$ dx, giving your answer to 5 decimal places. **(4)**

(c) By considering the size of the next term in the integrated series, explain why this estimate is accurate to at least 5 decimal places. **(2)**

(Total for Question 3 is 9 marks)

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4 $f(x) = \cos(x)\cos(x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 4 is 6 marks)

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Mark scheme · 1.11 Maclaurin Series: 30-minute Pack

Question 1

(a) substitutes u = 2x into 1 + u + $u^2/2!$ + $u^3/3!$ **M1**

(a) 1 + 2x + $4x^2/2$ + $8x^3/6$ unsimplified **A1**

(a) e^{2x} = 1 + 2x + $2x^2$ + $(4/3)x^3$ cao **A1**

(a) Answer: e^{2x} = 1 + 2x + $2x^2$ + $(4/3)x^3$

(b) substitutes u = 3x into u - $u^3/3!$ + $u^5/5!$ **M1**

(b) 3x - $27x^3/6$ + $243x^5/120$ unsimplified **A1**

(b) coefficient of x^3 simplified to -(9/2) cao **A1**

(b) sin(3x) = 3x - $(9/2)x^3$ + $(81/40)x^5$ cao **A1**

(b) Answer: sin(3x) = 3x - $(9/2)x^3$ + $(81/40)x^5$

Question 2

(a) writes e^x - 1 - x = $x^2/2$ + $x^3/6$ + $x^4/24$ + ... using the standard series **M1**

(a) divides by x^2 to get 1/2 + x/6 + $x^2/24$ + ... **A1**

(a) cao: limit = 1/2 **A1**

(a) Answer: 1/2

(b) writes 1 - cos x = $x^2/2$ - $x^4/24$ + $x^6/720$ - ... using the standard series **M1**

(b) subtracts $x^2/2$ to leave -$x^4/24$ + $x^6/720$ - ... **M1**

(b) divides by x^4 to get -1/24 + $x^2/720$ - ... **A1**

(b) cao: limit = -1/24 **A1**

(b) Answer: -1/24

Question 3

(a) substitutes u = x^2 into u - $u^3/3!$ + $u^5/5!$ **M1**

(a) u^3 = x^6 term gives -$x^6/6$ **A1**

(a) cso: u^5 = x^{10} term gives +$x^{10}/120$, confirming x^2 - $x^6/6$ + $x^{10}/120$ **A1**

(a) Answer: sin(x^2) = x^2 - $x^6/6$ + $x^{10}/120$

(b) integrates term by term to get $x^3/3$ - $x^7/42$ + $x^{11}/1320$ (+ C, C = 0) **M1**

(b) substitutes the limits x = 0.5 and x = 0 **M1**

(b) evaluates the three terms as awrt 0.04167, 0.00019, 0.0000004 **A1**

(b) cao: integral is approximately 0.04148 (5 dp) **A1**

(b) Answer: Integral from 0 to 0.5 of sin(x^2) dx is approximately 0.04148

(c) identifies that the next term involves $x^{15}/75600$ (from integrating -$x^{14}/5040$) **B1**

(c) evaluates this term at x = 0.5 as approximately 4 x 10^{-10}, which is far smaller than 0.000005, so the truncation error cannot affect the 5th decimal place **B1**

(c) Answer: The next term is approximately 4 x 10^{-10} at x = 0.5, far below the rounding threshold of 0.000005, so the estimate is accurate to at least 5 dp

Question 4

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) 1 - x^2 + $(1/3)x^4$ - $(2/45)x^6$ **A1**

(a) Answer: 1 - x^2 + $(1/3)x^4$ - $(2/45)x^6$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.99003 **A1**

(b) Answer: 0.99003

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