1.11 Maclaurin Series: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The function $f(x) = e^x$ is to be expanded as a Maclaurin series.

(a) By finding successive derivatives of $f(x) = e^x$, evaluate f(0), f'(0), f''(0), f'''(0) and f''''(0). **(2)**

(b) Hence find the Maclaurin series for e^x, in ascending powers of x, up to and including the term in x^4. **(3)**

(c) State the general term of the series, and give the range of values of x for which the Maclaurin series for e^x is valid. **(2)**

(Total for Question 1 is 7 marks)

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2 The function $f(x) = \ln(1+x)$ is to be expanded as a Maclaurin series.

(a) Given that $f(x) = \ln(1+x)$, show that $f''(0) = -1$ and find the value of f'''(0). **(3)**

(b) Given further that $f''''(0) = -6$, find the Maclaurin series for $\ln(1+x)$, in ascending powers of x, up to and including the term in x^4. **(3)**

(c) State the range of values of x for which this series is valid. **(1)**

(Total for Question 2 is 7 marks)

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3 $f(x) = \sin(x) \ln(1 + 3x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 3 is 6 marks)

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4 Let $f(x) = \ln(1 + \sin x)$.

(a) Show that $f''(x) = -1/(1 + \sin x)$. **(4)**

(b) Hence find the value of $f'''(0)$. **(2)**

(c) Show that the Maclaurin series for $\ln(1+\sin x)$, up to and including the term in x^3, is $x - x^2/2 + x^3/6$. **(3)**

(Total for Question 4 is 9 marks)

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5 Let $g(x) = 1/(1+x^2)$.

(a) Write down the series expansion of $1/(1+x^2)$, as a series in ascending powers of x, up to and including the term in x^6, and state the range of values of x for which the expansion is valid. **(3)**

(b) By integrating the series in part (a) term by term, and using the fact that $\arctan(0) = 0$, show that $\arctan x = x - x^3/3 + x^5/5 - x^7/7 + \ldots$. **(3)**

(c) Using the first four non-zero terms of this series, with $x = 1/\sqrt{3}$, find an estimate for the value of π, giving your answer to 3 decimal places. Hence find the percentage error in this estimate compared with the value of π given by your calculator, giving your answer to 2 significant figures. **(4)**

(Total for Question 5 is 10 marks)

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6 A function y(x) satisfies the differential equation $d^2y/dx^2 = x(dy/dx) + 2y$, with $y = 1$ and $dy/dx = 0$ when $x = 0$.

(a) By repeated differentiation of the given equation, find the values of d^3y/dx^3, d^4y/dx^4 and d^5y/dx^5 when $x = 0$. Show your method clearly. **(6)**

(b) Hence find the Maclaurin series for y, in ascending powers of x, up to and including the term in x^4. **(3)**

(c) Use your series to find an estimate, to 4 decimal places, for the value of y when $x = 0.3$. **(2)**

(Total for Question 6 is 11 marks)

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Mark scheme · 1.11 Maclaurin Series: Test Pack

Question 1

(a) recognises $f^{n}(x) = e^{x}$ for every derivative oe **B1**

(a) all five values correctly stated as 1 cao **B1**

(a) Answer: f(0) = f'(0) = f''(0) = f'''(0) = f''''(0) = 1

(b) uses $f(x) = f(0) + f'(0)x + f''(0)x^2/2! + f'''(0)x^3/3! + f''''(0)x^4/4!$ oe **M1**

(b) correct series with the values from part (a) substituted, unsimplified **A1**

(b) $e^x = 1 + x + x^2/2 + x^3/6 + x^4/24$ cao **A1**

(b) Answer: $e^x = 1 + x + x^2/2 + x^3/6 + x^4/24$

(c) general term $x^n/n!$ oe **B1**

(c) valid for all real x cao **B1**

(c) Answer: General term $x^n/n!$; valid for all real x

Question 2

(a) $f'(x) = (1+x)^{-1}$, $f''(x) = -(1+x)^{-2}$, $f'''(x) = 2(1+x)^{-3}$ oe **M1**

(a) cso: f''(0) = -1 as required **A1**

(a) f'''(0) = 2 cao **A1**

(a) Answer: f''(0) = -1, f'''(0) = 2

(b) uses $f(x) = f(0) + f'(0)x + f''(0)x^2/2! + f'''(0)x^3/3! + f''''(0)x^4/4!$ with f(0) = 0, f'(0) = 1 **M1**

(b) correct series with values substituted, unsimplified **A1**

(b) $\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4$ cao **A1**

(b) Answer: $\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4$

(c) $-1 < x \leq 1$ oe cao **B1**

(c) Answer: $-1 < x \leq 1$

Question 3

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $3x^2 - (9/2)x^3 + (17/2)x^4 - (39/2)x^5$ **A1**

(a) Answer: $3x^2 - (9/2)x^3 + (17/2)x^4 - (39/2)x^5$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.02616 **A1**

(b) Answer: 0.02616

Question 4

(a) differentiates f(x) using the chain rule to get $f'(x) = \cos x/(1+\sin x)$ **M1**

(a) attempts to differentiate f'(x) using the quotient rule **M1**

(a) numerator simplifies to $-(\sin x + \sin^2 x + \cos^2 x) = -(1 + \sin x)$ using $\sin^2 x + \cos^2 x = 1$ **A1**

(a) cso: $f''(x) = -(1+\sin x)/(1+\sin x)^2 = -1/(1+\sin x)$ as required **A1**

(a) Answer: f''(x) = -1/(1+sin x)

(b) differentiates $-(1+\sin x)^{-1}$ using the chain rule to get $f'''(x) = \cos x/(1+\sin x)^2$ **M1**

(b) f'''(0) = 1/1 = 1 cao **A1**

(b) Answer: f'''(0) = 1

(c) substitutes f(0)=0, f'(0)=1, f''(0)=-1, f'''(0)=1 into the Maclaurin formula **M1**

(c) $0 + 1(x) + (-1)x^2/2! + 1(x^3)/3!$ unsimplified **A1**

(c) cso: $x - x^2/2 + x^3/6$ as required **A1**

(c) Answer: $\ln(1+\sin x) = x - x^2/2 + x^3/6$

Question 5

(a) recognises $1/(1+x^2)$ as a geometric series with first term 1 and common ratio $-x^2$ **B1**

(a) $1 - x^2 + x^4 - x^6$ cao **B1**

(a) valid for $-1 < x < 1$ oe **B1**

(a) Answer: $1/(1+x^2) = 1 - x^2 + x^4 - x^6$; valid for $-1 < x < 1$

(b) uses $d/dx(\arctan x) = 1/(1+x^2)$ and integrates each term of the series in part (a) **M1**

(b) obtains $x - x^3/3 + x^5/5 - x^7/7$ (+ C) **A1**

(b) cso: C = 0 since arctan(0) = 0, giving the printed result **A1**

(b) Answer: $\arctan x = x - x^3/3 + x^5/5 - x^7/7$

(c) uses $\arctan(1/\sqrt{3}) = \pi/6$ and substitutes $x = 1/\sqrt{3}$ into the series **M1**

(c) evaluates the sum of the first four non-zero terms (awrt 0.523) **M1**

(c) π is approximately 6 x 0.52298 = 3.138 (3 dp) awrt **A1**

(c) ft: percentage error = $|\pi(\text{calc}) - \text{estimate}|/\pi(\text{calc}) \times 100$ = 0.12% (2 sf) awrt **A1**

(c) Answer: π is approximately 3.138; percentage error is approximately 0.12%

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Question 6

(a) differentiates $y'' = xy' + 2y$ using the product rule to get $y''' = 3y' + xy''$ **M1**

(a) substitutes x=0, y'=0, y''=2 to get $y'''(0) = 0$ **A1**

(a) differentiates again to get $y'''' = 4y'' + xy'''$ **M1**

(a) substitutes x=0, y''=2, y'''=0 to get $y''''(0) = 8$ **A1**

(a) differentiates again to get $y''''' = 5y''' + xy''''$ **M1**

(a) substitutes x=0, y'''=0, y''''=8 to get $y'''''(0) = 0$ **A1**

(a) Answer: $y'''(0) = 0$, $y''''(0) = 8$, $y'''''(0) = 0$

(b) substitutes y(0)=1, y'(0)=0, y''(0)=2, y'''(0)=0, y''''(0)=8 into the Maclaurin formula **M1**

(b) $1 + 0 + 2x^2/2! + 0 + 8x^4/4!$ unsimplified **A1**

(b) cao: $y = 1 + x^2 + x^4/3$ **A1**

(b) Answer: $y = 1 + x^2 + x^4/3$

(c) substitutes x = 0.3 into $y = 1 + x^2 + x^4/3$ **M1**

(c) cao: y is approximately 1.0927 (4 dp) **A1**

(c) Answer: y(0.3) is approximately 1.0927

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