1.11 Maclaurin Series

EDEXCEL 9FM0 · Calculator allowed · about 220 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The function f(x) = cos x is to be expanded as a Maclaurin series.

(a) By finding successive derivatives of f(x) = cos x, find the values of f(0), f'(0), f''(0), f'''(0), f''''(0), f'''''(0) and f''''''(0). **(2)**

(b) Hence show that the Maclaurin series for cos x, up to and including the term in x^6, is $1 - x^2/2 + x^4/24 - x^6/720$. **(3)**

(c) Write down the general term of the series, and state the values of x for which the expansion is valid. **(2)**

(Total for Question 1 is 7 marks)

2 You may use, without proof, the standard Maclaurin series $e^x = 1 + x + x^2/2! + x^3/3! + ...$ and $\sin x = x - x^3/3! + x^5/5! - ...$, both valid for all real x.

(a) By substituting into the series for e^x, find the Maclaurin series for e^{2x}, in ascending powers of x, up to and including the term in x^3. **(3)**

(b) By substituting into the series for sin x, find the Maclaurin series for sin(3x), in ascending powers of x, up to and including the term in x^5, giving each coefficient as an exact fraction. **(4)**

(Total for Question 2 is 7 marks)

3 Maclaurin series can be used to evaluate limits that are not directly computable by substitution.

(a) By using the Maclaurin series for e^x, find $\lim(x\to 0)$ of $(e^x - 1 - x)/x^2$. **(3)**

(b) By using the Maclaurin series for cos x, find $\lim(x\to 0)$ of $(1 - \cos x - x^2/2)/x^4$. **(4)**

(Total for Question 3 is 7 marks)

4 Consider the function $f(x) = e^x \sin x$.

(a) Write down the standard Maclaurin series for e^x and for $\sin x$, in each case up to and including the term in x^4. **(2)**

(b) By multiplying these two series together, find the Maclaurin series for $e^x \sin x$, in ascending powers of x, up to and including the term in x^4, giving each coefficient as an exact fraction (or integer). **(5)**

(Total for Question 4 is 7 marks)

5 Consider the function $f(x) = e^x \sqrt{1+x}$.

(a) Write down the binomial expansion of $(1+x)^{1/2}$, in ascending powers of x, up to and including the term in x^3, simplifying each coefficient. **(3)**

(b) Using the standard Maclaurin series for e^x, find the Maclaurin series for $f(x) = e^x \sqrt{1+x}$, in ascending powers of x, up to and including the term in x^3, giving each coefficient as a fraction in its simplest form. **(5)**

(Total for Question 5 is 8 marks)

6 Consider the function $h(x) = \sin(x^2)$.

(a) Using the standard Maclaurin series for sin x, show that the Maclaurin series for $\sin(x^2)$, up to and including the term in x^{10}, is $x^2 - x^6/6 + x^{10}/120$. **(3)**

(b) By integrating this series term by term, find an estimate for the integral from 0 to 0.5 of $\sin(x^2)\,dx$, giving your answer to 5 decimal places. **(4)**

(c) By considering the size of the next term in the integrated series, explain why this estimate is accurate to at least 5 decimal places. **(2)**

(Total for Question 6 is 9 marks)

7 $f(x) = \sqrt{1 + \sin(x)}$.

(a) Find the Maclaurin series for f(x) up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of f(0), f'(0) and f''(0). **(3)**

(Total for Question 7 is 7 marks)

8 $f(x) = \cos(x)\cos(x)$.

(a) Use the standard Maclaurin series to find the series expansion of $f(x)$ in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for $f(0.1)$, to 5 decimal places, using these four terms. **(2)**

(Total for Question 8 is 6 marks)

9 $f(x) = e^{\sin(2x)}$.

(a) Find the Maclaurin series for f(x) up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of f(0), f'(0) and f''(0). **(3)**

(Total for Question 9 is 7 marks)

10 $f(x) = e^{3x}/(1 + 3x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 10 is 6 marks)

11 $f(x) = \cos(\sin(2x))$.

(a) Find the Maclaurin series for $f(x)$ up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of $f(0)$, $f'(0)$ and $f''(0)$. **(3)**

(Total for Question 11 is 7 marks)

12 This question is about limits found using Maclaurin series.

(a) Use the Maclaurin series for e^x to find the limit as x tends to 0 of $(e^x - 1 - x)/x^2$. **(3)**

(Total for Question 12 is 3 marks)

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13 $f(x) = e^{2x}$.

(a) Find the Maclaurin series for f(x) up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of f(0), f'(0) and f''(0). **(3)**

(Total for Question 13 is 7 marks)

14 $f(x) = \sin(2x) \ln(1 + 3x)$.

(a) Use the standard Maclaurin series to find the series expansion of $f(x)$ in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for $f(0.1)$, to 5 decimal places, using these four terms. **(2)**

(Total for Question 14 is 6 marks)

15 $f(x) = e^{\sin(x)}$.

(a) Find the Maclaurin series for f(x) up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of f(0), f'(0) and f''(0). **(2)**

(Total for Question 15 is 6 marks)

16 $f(x) = e^{2x}/(1 + 2x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 16 is 6 marks)

17 $f(x) = \sin(2x) \ln(1 + 2x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 17 is 6 marks)

18 This question is about limits found using Maclaurin series.

(a) Find the limit as x tends to 0 of $(\cos(3x) - 1)/x^2$ using a Maclaurin series. **(3)**

(Total for Question 18 is 3 marks)

19 This question is about limits found using Maclaurin series.

(a) Find the limit as x tends to 0 of $(e^x - \cos(4x) - x)/x^2$ using Maclaurin series. **(4)**

(Total for Question 19 is 4 marks)

20 This question is about limits found using Maclaurin series.

(a) Find the limit as x tends to 0 of $(\cos(x) - 1)/x^2$ using a Maclaurin series. **(3)**

(Total for Question 20 is 3 marks)

21 This question is about limits found using Maclaurin series.

(a) Find the limit as x tends to 0 of $(\ln(1 + x) - x)/x^2$ using a Maclaurin series. **(3)**

(Total for Question 21 is 3 marks)

22 $f(x) = \sqrt{1 + \sin(2x)}$.

(a) Find the Maclaurin series for f(x) up to and including the term in x^4, by composing standard series. **(4)**

(b) Hence find the value of f(0), f'(0) and f''(0). **(2)**

(Total for Question 22 is 6 marks)

23 $f(x) = e^{2x} \ln(1 + 3x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 23 is 6 marks)

24 $f(x) = e^{2x}/(1 + x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 24 is 6 marks)

25 $f(x) = e^x/(1 + 3x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 25 is 6 marks)

26 This question is about limits found using Maclaurin series.

(a) Find the limit as x tends to 0 of $(\sin(x) - x)/x^3$ using a Maclaurin series. **(3)**

(Total for Question 26 is 3 marks)

27 A curve satisfies the differential equation $dy/dx = x + y^2$ and passes through the point (0, 1).

(a) Show that $dy/dx = 1$ when $x = 0$, and find the value of d^2y/dx^2 when $x = 0$. **(4)**

(b) Find the value of d^3y/dx^3 when $x = 0$. **(3)**

(c) Hence find the series solution for y, in ascending powers of x, up to and including the term in x^3. **(2)**

(Total for Question 27 is 9 marks)

28 Let $f(x) = \sec x$.

(a) Show that $f'(x) = \sec x \tan x$, and hence show that $f''(x) = \sec x \tan^2 x + \sec^3 x$. **(5)**
State the value of $f''(0)$.

(b) Given further that $f'''(x) = \sec x \tan^3 x + 5 \sec^3 x \tan x$, show that $f''''(0) = 5$. **(4)**

(c) Hence show that the Maclaurin series for sec x, up to and including the term in x^4, is $1 + x^2/2 + 5x^4/24$. **(2)**

(Total for Question 28 is 11 marks)

Mark scheme · 1.11 Maclaurin Series

Question 1

(a) derivatives cycle cos x, -sin x, -cos x, sin x with correct signs oe **B1**

(a) all seven values correctly stated as 1, 0, -1, 0, 1, 0, -1 **B1**

(a) Answer: f(0)=1, f'(0)=0, f''(0)=-1, f'''(0)=0, f''''(0)=1, f'''''(0)=0, f''''''(0)=-1

(b) uses $f(x) = f(0) + f'(0)x + f''(0)x^2/2! + ... + f''''''(0)x^6/6!$ oe **M1**

(b) correct series with values substituted, odd-power terms vanish **A1**

(b) cso: $1 - x^2/2 + x^4/24 - x^6/720$ as required **A1**

(b) Answer: cos x = $1 - x^2/2 + x^4/24 - x^6/720$

(c) general term $(-1)^n x^{2n}/(2n)!$ oe **B1**

(c) valid for all real x cao **B1**

(c) Answer: General term $(-1)^n x^{2n}/(2n)!$; valid for all real x

Question 2

(a) substitutes u = 2x into $1 + u + u^2/2! + u^3/3!$ **M1**

(a) $1 + 2x + 4x^2/2 + 8x^3/6$ unsimplified **A1**

(a) $e^{2x} = 1 + 2x + 2x^2 + (4/3)x^3$ cao **A1**

(a) Answer: $e^{2x} = 1 + 2x + 2x^2 + (4/3)x^3$

(b) substitutes u = 3x into $u - u^3/3! + u^5/5!$ **M1**

(b) $3x - 27x^3/6 + 243x^5/120$ unsimplified **A1**

(b) coefficient of x^3 simplified to -(9/2) cao **A1**

(b) $\sin(3x) = 3x - (9/2)x^3 + (81/40)x^5$ cao **A1**

(b) Answer: $\sin(3x) = 3x - (9/2)x^3 + (81/40)x^5$

Question 3

(a) writes $e^x - 1 - x = x^2/2 + x^3/6 + x^4/24 + ...$ using the standard series **M1**

(a) divides by x^2 to get $1/2 + x/6 + x^2/24 + ...$ **A1**

(a) cao: limit = 1/2 **A1**

(a) Answer: 1/2

(b) writes $1 - \cos x = x^2/2 - x^4/24 + x^6/720 - ...$ using the standard series **M1**

(b) subtracts $x^2/2$ to leave $-x^4/24 + x^6/720 - ...$ **M1**

(b) divides by x^4 to get $-1/24 + x^2/720 - ...$ **A1**

(b) cao: limit = -1/24 **A1**

(b) Answer: -1/24

Question 4

(a) $e^x = 1 + x + x^2/2 + x^3/6 + x^4/24$ **B1**

(a) $\sin x = x - x^3/6 (+ 0x^4)$ **B1**

(a) Answer: $e^x = 1+x+x^2/2+x^3/6+x^4/24$; $\sin x = x - x^3/6$

(b) attempts to multiply the two series, collecting terms of equal power up to x^4 **M1**

(b) coefficient of x is 1 **A1**

(b) coefficient of x^2 is 1 **A1**

(b) coefficient of x^3 is 1/3 **A1**

(b) coefficient of x^4 is 0, so $e^x \sin x = x + x^2 + x^3/3 (+ 0x^4)$ cao **A1**

(b) Answer: $e^x \sin x = x + x^2 + x^3/3$

Question 5

(a) uses $(1+x)^n = 1 + nx + n(n-1)x^2/2! + n(n-1)(n-2)x^3/3!$ with n = 1/2 **M1**

(a) coefficient of x^2 simplifies to -1/8 **A1**

(a) coefficient of x^3 simplifies to 1/16, giving $1 + x/2 - x^2/8 + x^3/16$ cao **A1**

(a) Answer: $(1+x)^{1/2} = 1 + x/2 - x^2/8 + x^3/16$

(b) attempts to multiply the series for e^x and for $(1+x)^{1/2}$, collecting terms up to x^3 **M1**

(b) constant term is 1 **A1**

(b) coefficient of x is 3/2 **A1**

(b) coefficient of x^2 is 7/8 **A1**

(b) coefficient of x^3 is 17/48 cao **A1**

(b) Answer: $e^x \sqrt{1+x} = 1 + (3/2)x + (7/8)x^2 + (17/48)x^3$

Question 6

(a) substitutes $u = x^2$ into $u - u^3/3! + u^5/5!$ **M1**

(a) $u^3 = x^6$ term gives $-x^6/6$ **A1**

(a) cso: $u^5 = x^{10}$ term gives $+x^{10}/120$, confirming $x^2 - x^6/6 + x^{10}/120$ **A1**

(a) Answer: $\sin(x^2) = x^2 - x^6/6 + x^{10}/120$

(b) integrates term by term to get $x^3/3 - x^7/42 + x^{11}/1320$ (+ C, C = 0) **M1**

(b) substitutes the limits x = 0.5 and x = 0 **M1**

(b) evaluates the three terms as awrt 0.04167, 0.00019, 0.0000004 **A1**

(b) cao: integral is approximately 0.04148 (5 dp) **A1**

(b) Answer: Integral from 0 to 0.5 of $\sin(x^2)$ dx is approximately 0.04148

(c) identifies that the next term involves $x^{15}/75600$ (from integrating $-x^{14}/5040$) **B1**

(c) evaluates this term at x = 0.5 as approximately 4×10^{-10}, which is far smaller than 0.000005, so the truncation error cannot affect the 5th decimal place **B1**

(c) Answer: The next term is approximately 4×10^{-10} at x = 0.5, far below the rounding threshold of 0.000005, so the estimate is accurate to at least 5 dp

Question 7

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 + (1/2)x - (1/8)x^2 - (1/48)x^3 + (1/384)x^4$ **A1**

(a) Answer: $1 + (1/2)x - (1/8)x^2 - (1/48)x^3 + (1/384)x^4$

(b) f(0) = 1 **B1**

(b) f'(0) = 1/2 **B1**

(b) f''(0) = 2 x coefficient of x^2 = -1/4 **B1**

(b) Answer: f(0) = 1, f'(0) = 1/2, f''(0) = -1/4

Question 8

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 - x^2 + (1/3)x^4 - (2/45)x^6$ **A1**

(a) Answer: $1 - x^2 + (1/3)x^4 - (2/45)x^6$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.99003 **A1**

(b) Answer: 0.99003

Question 9

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 + 2x + 2x^2 - 2x^4$ **A1**

(a) Answer: $1 + 2x + 2x^2 - 2x^4$

(b) f(0) = 1 **B1**

(b) f'(0) = 2 **B1**

(b) f''(0) = 2 x coefficient of x^2 = 4 **B1**

(b) Answer: f(0) = 1, f'(0) = 2, f''(0) = 4

Question 10

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 + (9/2)x^2 - 9x^3 + (243/8)x^4$ **A1**

(a) Answer: $1 + (9/2)x^2 - 9x^3 + (243/8)x^4$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 1.03904 **A1**

(b) Answer: 1.03904

Question 11

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 - 2x^2 + (10/3)x^4$ **A1**

(a) Answer: $1 - 2x^2 + (10/3)x^4$

(b) f(0) = 1 **B1**

(b) f'(0) = 0 **B1**

(b) f''(0) = 2 x coefficient of x^2 = -4 **B1**

(b) Answer: f(0) = 1, f'(0) = 0, f''(0) = -4

Question 12

(a) $e^x = 1 + x + (x)^2/2 + ...$ **M1**

(a) subtracts and divides by x^2 **M1**

(a) 1/2 **A1**

(a) Answer: 1/2

Question 13

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 + 2x + 2x^2 + (4/3)x^3 + (2/3)x^4$ **A1**

(a) Answer: $1 + 2x + 2x^2 + (4/3)x^3 + (2/3)x^4$

(b) f(0) = 1 **B1**

(b) f'(0) = 2 **B1**

(b) f''(0) = 2 x coefficient of x^2 = 4 **B1**

(b) Answer: f(0) = 1, f'(0) = 2, f''(0) = 4

Question 14

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $6x^2 - 9x^3 + 14x^4 - (69/2)x^5$ **A1**

(a) Answer: $6x^2 - 9x^3 + 14x^4 - (69/2)x^5$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.05206 **A1**

(b) Answer: 0.05206

Question 15

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 + x + (1/2)x^2 - (1/8)x^4$ **A1**

(a) Answer: $1 + x + (1/2)x^2 - (1/8)x^4$

(b) f'(0) = 1 **B1**

(b) f''(0) = 2 x coefficient of x^2 = 1 **B1**

(b) Answer: f(0) = 1, f'(0) = 1, f''(0) = 1

Question 16

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 + 2x^2 - (8/3)x^3 + 6x^4$ **A1**

(a) Answer: $1 + 2x^2 - (8/3)x^3 + 6x^4$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 1.01793 **A1**

(b) Answer: 1.01793

Question 17

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $4x^2 - 4x^3 + (8/3)x^4 - (16/3)x^5$ **A1**

(a) Answer: $4x^2 - 4x^3 + (8/3)x^4 - (16/3)x^5$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.03621 **A1**

(b) Answer: 0.03621

Question 18

(a) $\cos(3x) = 1 - (3x)^2/2 + ...$ **M1**

(a) subtracts 1 and divides by x^2 **M1**

(a) -9/2 **A1**

(a) Answer: -9/2

Question 19

(a) $e^x = 1 + x + (x)^2/2 + ...$ and $\cos(4x) = 1 - (4x)^2/2 + ...$ **M1**

(a) combines: constant and x terms cancel **M1**

(a) coefficient of x^2 is 1/2 + 8 **A1**

(a) 17/2 **A1**

(a) Answer: 17/2

Question 20

(a) $\cos(x) = 1 - (x)^2/2 + ...$ **M1**

(a) subtracts 1 and divides by x^2 **M1**

(a) -1/2 **A1**

(a) Answer: -1/2

Question 21

(a) $\ln(1 + x) = x - (x)^2/2 + ...$ **M1**

(a) subtracts and divides by x^2 **M1**

(a) -1/2 **A1**

(a) Answer: -1/2

Question 22

(a) writes the standard series for the outer function in terms of the inner function **M1**

(a) substitutes the series of the inner function and expands, keeping terms up to x^4 **M1**

(a) terms up to x^2 correct **A1**

(a) $1 + x - (1/2)x^2 - (1/6)x^3 + (1/24)x^4$ **A1**

(a) Answer: $1 + x - (1/2)x^2 - (1/6)x^3 + (1/24)x^4$

(b) f'(0) = 1 **B1**

(b) f''(0) = 2 x coefficient of x^2 = -1 **B1**

(b) Answer: f(0) = 1, f'(0) = 1, f''(0) = -1

Question 23

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $3x + (3/2)x^2 + 6x^3 - (29/4)x^4$ **A1**

(a) Answer: $3x + (3/2)x^2 + 6x^3 - (29/4)x^4$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.32028 **A1**

(b) Answer: 0.32028

Question 24

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 + x + x^2 + (1/3)x^3$ **A1**

(a) Answer: $1 + x + x^2 + (1/3)x^3$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 1.11033 **A1**

(b) Answer: 1.11033

Question 25

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 - 2x + (13/2)x^2 - (58/3)x^3$ **A1**

(a) Answer: $1 - 2x + (13/2)x^2 - (58/3)x^3$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.84567 **A1**

(b) Answer: 0.84567

Question 26

(a) $\sin(x) = x - (x)^3/6 + ...$ **M1**

(a) subtracts and divides by x^3 **M1**

(a) -1/6 **A1**

(a) Answer: -1/6

Question 27

(a) cso: $dy/dx = 0 + 1^2 = 1$ when x = 0 **B1**

(a) differentiates $dy/dx = x + y^2$ implicitly to get $d^2y/dx^2 = 1 + 2y(dy/dx)$ **M1**

(a) substitutes x=0, y=1, dy/dx=1 **A1**

(a) cao: $d^2y/dx^2 = 1 + 2(1)(1) = 3$ **A1**

(a) Answer: $dy/dx = 1$ and $d^2y/dx^2 = 3$ at x = 0

(b) differentiates $d^2y/dx^2 = 1 + 2y(dy/dx)$ to get $d^3y/dx^3 = 2(dy/dx)^2 + 2y(d^2y/dx^2)$ **M1**

(b) substitutes dy/dx=1, y=1, $d^2y/dx^2=3$ **A1**

(b) cao: $d^3y/dx^3 = 2(1)^2 + 2(1)(3) = 8$ **A1**

(b) Answer: $d^3y/dx^3 = 8$ at x = 0

(c) substitutes y(0)=1, y'(0)=1, y''(0)=3, y'''(0)=8 into the Maclaurin formula **M1**

(c) cao: $y = 1 + x + (3/2)x^2 + (4/3)x^3$ **A1**

(c) Answer: $y = 1 + x + (3/2)x^2 + (4/3)x^3$

Question 28

(a) differentiates $f(x) = (\cos x)^{-1}$ using the chain rule **M1**

(a) cso: $f'(x) = \sin x (\cos x)^{-2} = \sec x \tan x$ **A1**

(a) differentiates $f'(x) = \sec x \tan x$ using the product rule: $\sec x \tan x \cdot \tan x + \sec x \cdot \sec^2 x$ **M1**

(a) cso: $f''(x) = \sec x \tan^2 x + \sec^3 x$ as required **A1**

(a) $f''(0) = 0 + 1 = 1$ cao **B1**

(a) Answer: $f''(x) = \sec x \tan^2 x + \sec^3 x$, and $f''(0) = 1$

(b) differentiates $\sec x \tan^3 x$ using the product rule to get $\sec x \tan^4 x + 3\sec^3 x \tan^2 x$ **M1**

(b) differentiates $5\sec^3 x \tan x$ using the product rule to get $15\sec^3 x \tan^2 x + 5\sec^5 x$ **M1**

(b) combines to $f''''(x) = \sec x \tan^4 x + 18\sec^3 x \tan^2 x + 5\sec^5 x$ **A1**

(b) cso: substitutes $x=0$ ($\sec 0=1$, $\tan 0=0$) to get $f''''(0) = 0 + 0 + 5 = 5$ as required **A1**

(b) Answer: $f''''(0) = 5$

(c) substitutes $f(0)=1$, $f'(0)=0$, $f''(0)=1$, $f'''(0)=0$, $f''''(0)=5$ into the Maclaurin formula **M1**

(c) cso: $1 + 0 + x^2/2! + 0 + 5x^4/4! = 1 + x^2/2 + 5x^4/24$ as required **A1**

(c) Answer: $\sec x = 1 + x^2/2 + 5x^4/24$