1.12 Core Pure: Complex Numbers Depth: 30-minute Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Let $z = 1 + i$.

(a) Find the modulus and argument of z. **(2)**

(b) Hence use de Moivre's theorem to find z^8, giving your answer as a real integer. You must show your method. **(2)**

(Total for Question 1 is 4 marks)

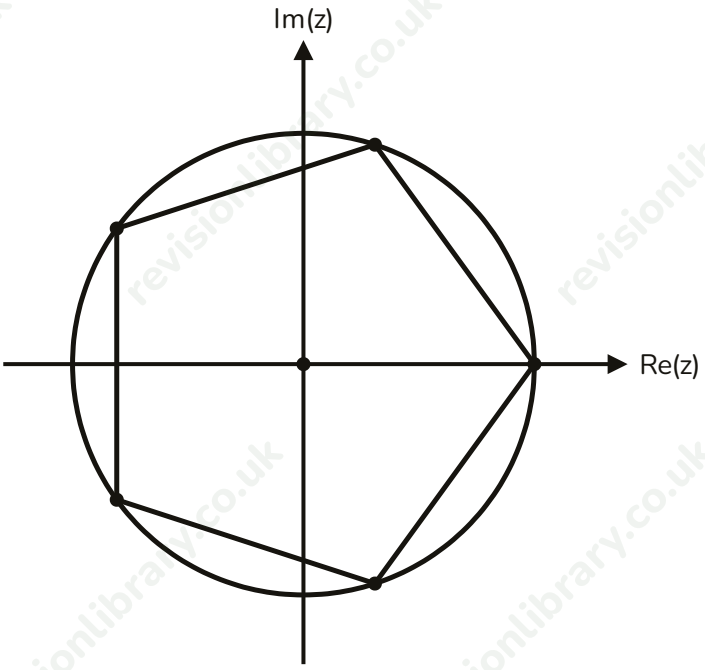
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2 The complex number $u = \cos(2*\pi/5) + i*\sin(2*\pi/5)$ is a fifth root of unity.

(a) Write down, in a similar form, the other four fifth roots of unity (i.e. the roots of $z^5 = 1$ other than u), for k = 0, 2, 3, 4. **(2)**

(b) Show that $1 + u + u^2 + u^3 + u^4 = 0$. **(3)**

(c) The five roots of $z^5 = 1$ are represented on an Argand diagram by points forming a regular pentagon inscribed in a circle of radius 1 centred at the origin. Show that the area of this pentagon is $(5/2)*\sin(2*\pi/5)$. **(5)**



(d) Hence find the area of the pentagon, giving your answer to 3 significant figures. **(2)**

(Total for Question 2 is 12 marks)

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3 The complex number z satisfies $|z - (-1 - 4i)| = 5$.

(a) Describe the locus of z geometrically, and write its equation in Cartesian form. **(4)**

(b) Find the greatest value of $|z|$ and the least value of $|z|$. **(3)**

(Total for Question 3 is 7 marks)

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4 Solve the equation $z^6 = 1$, giving each root in the form $a + bi$ with a and b exact.

(a) Write 1 in modulus-argument form. **(2)**

(b) Hence find all 6 roots of the equation. **(4)**

(Total for Question 4 is 6 marks)

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Mark scheme · 1.12 Core Pure: Complex Numbers Depth: 30-minute Pack

Question 1

(a) $|z| = \sqrt{1^2+1^2} = \sqrt{2}$ **B1**

(a) arg(z) = π/4, cao **B1**

(a) Answer: $|z| = \sqrt{2}$, arg(z) = π/4

(b) applies de Moivre's theorem: $z^8 = (\sqrt{2})^8*(\cos(8*\pi/4) + i*\sin(8*\pi/4))$, simplifying $(\sqrt{2})^8 = 16$ and 8*π/4 = 2*π **M1**

(b) $z^8 = 16$, cao **A1**

(b) Answer: $z^8 = 16$

Question 2

(a) roots given by cos(2*k*π/5) + i*sin(2*k*π/5) for k = 0, 2, 3, 4, or equivalent forms **B1**

(a) correctly identifies the other four roots as 1, u^2, u^3, u^4 (or equivalent forms), excluding the given root u **B1**

(a) Answer: The other four roots are 1, u^2, u^3, u^4, i.e. cos(2*k*π/5) + i*sin(2*k*π/5) for k = 0, 2, 3, 4

(b) recognises the sum as a geometric series with sum $(u^5 - 1)/(u - 1)$, valid since u is not equal to 1 **M1**

(b) $u^5 = 1$ (as u is a fifth root of unity), so the numerator is 0 **A1**

(b) hence the sum is 0, cso **A1**

(b) Answer: $1 + u + u^2 + u^3 + u^4 = 0$

(c) splits the pentagon into 5 congruent isosceles triangles from the centre, each with two sides of length 1 and included angle 2*π/5 **M1**

(c) area of one triangle = (1/2)*1*1*sin(2*π/5), using Area = (1/2)*a*b*sin(C) **M1**

(c) correct triangle area (1/2)*sin(2*π/5) **A1**

(c) total pentagon area = 5*(1/2)*sin(2*π/5) **M1**

(c) Area = (5/2)*sin(2*π/5), cso **A1**

(c) Answer: Area = (5/2)*sin(2*π/5)

(d) evaluates sin(2*π/5), approximately 0.951056 **M1**

(d) area = awrt 2.38 (3 s.f.) **A1**

(d) Answer: Area is approximately 2.38 (3 s.f.)

Question 3

(a) a circle with centre (-1, -4) **B1**

(a) radius 5 **B1**

(a) writes z = x + iy and uses $|z - w|^2$ **M1**

(a) $(x + 1)^2 + (y + 4)^2 = 25$ **A1**

(a) Answer: $(x + 1)^2 + (y + 4)^2 = 25$

(b) distance of the centre from the origin = $\sqrt{17}$ **M1**

(b) greatest = $\sqrt{17} + 5$ **A1**

(b) least = $|\sqrt{17} - 5|$ (the origin lies inside the circle when $\sqrt{17} < 5$) **A1**

(b) Answer: greatest $\sqrt{17} + 5 = 9.123$, least 0.8769

Question 4

(a) modulus and argument **M1**

(a) 1(cos 0 + i sin 0) (plus multiples of 360 degrees) **A1**

(a) Answer: 1(cos 0 + i sin 0)

(b) modulus 1 and arguments (0 + 360k)/6 degrees for k = 0, 1, ..., 5 **M1**

(b) arguments 0, 60, 120, 180, 240, 300 **A1**

(b) uses exact values of cosine and sine **M1**

(b) roots: 1; 1/2 + (1/2)$\sqrt{3}$i; -1/2 + (1/2)$\sqrt{3}$i; -1; -1/2 - (1/2)$\sqrt{3}$i; 1/2 - (1/2)$\sqrt{3}$i **A1**

(b) Answer: 1; 1/2 + (1/2)$\sqrt{3}$i; -1/2 + (1/2)$\sqrt{3}$i; -1; -1/2 - (1/2)$\sqrt{3}$i; 1/2 - (1/2)$\sqrt{3}$i

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