1.12 Core Pure: Complex Numbers Depth: Test Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 65 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Let $z = 3 - 3*\sqrt{3}*i$.

(a) Find the modulus of z. **(1)**

(b) Find the argument of z, giving your answer in radians in the range $-\pi < \theta \leq \pi$. **(2)**

(Total for Question 1 is 3 marks)

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2 The complex number z satisfies $|z + 2 - 3*i| = 4$.

(a) Show that the locus of z can be written as the Cartesian equation $(x + 2)^2 + (y - 3)^2 = 16$, where $z = x + i*y$. **(2)**

(b) State the centre and radius of this circle. **(2)**

(c) Determine, with a reason, whether the point representing $z = 2 + i$ lies inside, on, or outside this circle. **(1)**

(Total for Question 2 is 5 marks)

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3 Solve the equation $z^4 = 1296$, giving each root in the form a + bi with a and b exact.

(a) Write 1296 in modulus-argument form. **(2)**

(b) Hence find all 4 roots of the equation. **(4)**

(Total for Question 3 is 6 marks)

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4 The cubic equation $z^3 + pz^2 + qz - 40 = 0$, where p and q are real constants, has a root $3 + i$.

(a) Write down another complex root of the equation, with a reason. **(2)**

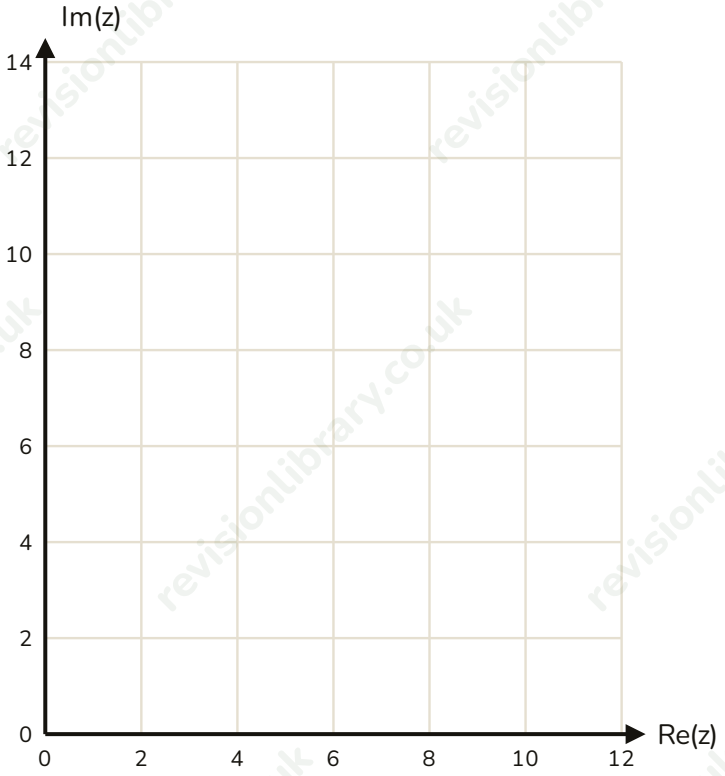
(b) Find the third root of the equation. **(3)**

(c) Find the values of p and q. **(4)**

(Total for Question 4 is 9 marks)

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5 The complex number z satisfies $|z - (6 + 8*i)| = 5$.



(a) Sketch the locus of z on an Argand diagram, indicating clearly the centre and radius of the circle. **(3)**

(b) Find the greatest and least possible values of $|z|$. **(4)**

(c) Find the range of possible values of $\arg(z)$, giving your answer in radians to 3 significant figures. **(5)**

(Total for Question 5 is 12 marks)

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6 The complex number z satisfies $|z - 9| = 2*|z|$.

(a) Show that the locus of z is a circle with Cartesian equation $(x + 3)^2 + y^2 = 36$, and state its centre and radius. **(6)**

(b) Find the greatest and least possible values of $|z|$ for points on this locus. **(4)**

(c) Given that w is the point on the locus with the greatest possible imaginary part, find w exactly, in the form $p + q*i$. **(4)**

(Total for Question 6 is 14 marks)

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Mark scheme · 1.12 Core Pure: Complex Numbers Depth: Test Pack

Question 1

(a) $|z| = \sqrt{3^2 + (3\sqrt{3})^2} = 6$, cao **B1**

(a) Answer: $|z| = 6$

(b) attempts $\arctan(3\sqrt{3}/3) = \arctan(\sqrt{3}) = \pi/3$ with correct fourth-quadrant sign consideration (real part positive, imaginary part negative) **M1**

(b) $\arg(z) = -\pi/3$, cao **A1**

(b) Answer: $\arg(z) = -\pi/3$

Question 2

(a) writes $|(x+2) + i(y-3)| = 4$ and squares both sides **M1**

(a) $(x+2)^2 + (y-3)^2 = 16$, cso **A1**

(a) Answer: $(x+2)^2 + (y-3)^2 = 16$

(b) centre = (-2, 3) **B1**

(b) radius = 4 **B1**

(b) Answer: Centre (-2, 3), radius 4

(c) correct conclusion (outside) with correct reason: $(2+2)^2 + (1-3)^2 = 20 > 16$ **B1**

(c) Answer: Outside the circle, since $(2+2)^2 + (1-3)^2 = 20 > 16$

Question 3

(a) modulus and argument **M1**

(a) $1296(\cos 0 + i \sin 0)$ (plus multiples of 360 degrees) **A1**

(a) Answer: $1296(\cos 0 + i \sin 0)$

(b) modulus 6 and arguments $(0 + 360k)/4$ degrees for k = 0, 1, ..., 3 **M1**

(b) arguments 0, 90, 180, 270 **A1**

(b) uses exact values of cosine and sine **M1**

(b) roots: 6; 6i; -6; -6i **A1**

(b) Answer: 6; 6i; -6; -6i

Question 4

(a) 3 - i **B1**

(a) complex roots of a polynomial with real coefficients occur in conjugate pairs **B1**

(a) Answer: 3 - i

(b) uses the product of the roots: $(\alpha)(\alpha^*)(r) = -(-40)$ **M1**

(b) $((3)^2 + 1^2)\, r = 40$ **A1**

(b) r = 4 **A1**

(b) Answer: 4

(c) uses sum of roots = -p **M1**

(c) p = -10 **A1**

(c) uses the sum of products of pairs = q **M1**

(c) q = 34 **A1**

(c) Answer: p = -10, q = 34

Question 5

(a) circle drawn, correctly positioned not passing through or enclosing the origin **B1**

(a) centre marked and labelled (6, 8) **B1**

(a) radius 5 indicated (e.g. circle shown touching x=1, x=11, y=3, y=13) **B1**

(a) Answer: Circle, centre (6, 8), radius 5

(b) finds the distance from the origin to the centre: $\sqrt{6^2+8^2} = 10$ **M1**

(b) correct reasoning that both extreme values of |z| occur on the line joining the origin to the centre, extended to meet the circle **B1**

(b) greatest |z| = 10 + 5 = 15 **A1**

(b) least |z| = 10 - 5 = 5, cao **A1**

(b) Answer: Greatest |z| = 15, least |z| = 5

(c) finds arg(centre) = arctan(8/6) **M1**

(c) finds the half-angle subtended at the origin by the circle: arcsin(radius/|OC|) = arcsin(5/10) = arcsin(0.5) = $\pi/6$ **M1**

(c) arctan(4/3) = 0.927 (awrt) **A1**

(c) range = arctan(4/3) - $\pi/6$ to arctan(4/3) + $\pi/6$ **M1**

(c) $0.404 \leq \arg(z) \leq 1.45$ (awrt, 3 s.f.) **A1**

(c) Answer: $0.404 \leq \arg(z) \leq 1.45$ (radians, 3 s.f.)

Question 6

(a) squares both sides: $(x-9)^2 + y^2 = 4*(x^2+y^2)$ **M1**

(a) expands both sides: $x^2-18x+81+y^2 = 4x^2+4y^2$ **M1**

(a) rearranges to $x^2 + y^2 + 6x - 27 = 0$ **M1**

(a) completes the square: $(x+3)^2 + y^2 = 36$, cso **A1**

(a) centre (-3, 0) **B1**

(a) radius 6 **B1**

(a) Answer: $(x+3)^2 + y^2 = 36$; centre (-3, 0); radius 6

(b) distance from the origin to the centre = $\sqrt{(-3)^2+0^2} = 3$, noting this is less than the radius 6, so the origin lies inside the circle **M1**

(b) least |z| = 6 - 3 = 3 **A1**

(b) greatest |z| = 6 + 3 = 9 **A1**

(b) both requested extreme values correctly identified as least 3 and greatest 9; identifying the corresponding points z = 3 and z = -9 is optional **B1**

(b) Answer: Least |z| = 3 (at z = 3); greatest |z| = 9 (at z = -9)

(c) recognises that the point of greatest imaginary part on a circle of centre (a,b) and radius r is (a, b+r) **M1**

(c) correct reasoning: the topmost point of the circle lies directly above the centre **B1**

(c) coordinates (-3, 6) **A1**

(c) w = -3 + 6*i, cao **A1**

(c) Answer: w = -3 + 6*i

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