1.13 Core Pure: Matrices and Transformations Depth: 1-hour Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The matrices A and B are given by A = ((5, -2), (3, 1)) and B = ((0, 0), (-1, 1)), where each matrix is written row by row.

(a) Find AB. **(2)**

(b) Find det A and, given that A is non-singular, find A^{-1}. **(4)**

(c) Use A^{-1} to solve the equations 5x - 2y = 17 and 3x + y = 8. **(3)**

(Total for Question 1 is 9 marks)

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2 The transformation T_1 is a reflection in the line y = x. The transformation T_2 is a stretch with scale factor 3 parallel to the y axis.

(a) Write down the 2 x 2 matrices M_1 and M_2 representing T_1 and T_2. **(2)**

(b) Find the matrix representing the transformation T_1 followed by T_2. **(2)**

(c) Find the image of the point (-2, -4) under T_1 followed by T_2. **(2)**

(d) A triangle of area 6 is transformed by T_1 followed by T_2. Find the area of its image. **(2)**

(Total for Question 2 is 8 marks)

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3 The matrix M is defined by M = [[4,7],[1,2]].

(a) Find det(M). **(2)**

(b) Find M^{-1}. **(2)**

(c) Hence use a matrix method to solve the simultaneous equations 4x + 7y = 15 and x + 2y = 4. **(3)**

(Total for Question 3 is 7 marks)

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4 The matrix A is given by A = ((k, -3, -2), (-1, 3, 0), (0, 3, 2)), written row by row, where k is a constant.

(a) Show that det A = 6k. **(3)**

(b) Find the value of k for which A is singular. **(2)**

(c) State the value of det A when k = 0, and hence state whether A is invertible when k = 0. **(2)**

(Total for Question 4 is 7 marks)

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5 The matrix A = [[2,a],[b,3]], where a and b are positive constants.

(a) Show that A^2 = [[4+ab,5a],[5b,ab+9]]. **(3)**

(b) Given that A^2 = [[7,15],[5,12]], find the values of a and b. **(5)**

(Total for Question 5 is 8 marks)

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6 The matrix Q is defined by Q = [[2k,1],[3,k+4]], where k is a constant.

(a) Given that y = x is an invariant line of the transformation represented by Q, show that k = 6. **(4)**

(b) Using this value of k, find the equation of the other invariant line of the transformation that passes through the origin. **(6)**

(Total for Question 6 is 10 marks)

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Mark scheme · 1.13 Core Pure: Matrices and Transformations Depth: 1-hour Pack

Question 1

(a) multiplies rows of A by columns of B **M1**

(a) ((2, -2), (-1, 1)) **A1**

(a) Answer: ((2, -2), (-1, 1))

(b) det A = ad - bc **M1**

(b) det A = 11 **A1**

(b) uses the inverse formula (1/det)((d, -b), (-c, a)) **M1**

(b) A^{-1} = ((1/11, 2/11), (-3/11, 5/11)) **A1**

(b) Answer: det A = 11, A^{-1} = ((1/11, 2/11), (-3/11, 5/11))

(c) writes as a matrix equation A(x, y) = (b1, b2) and multiplies by A^{-1} **M1**

(c) x = 3 **A1**

(c) y = -1 **A1**

(c) Answer: x = 3, y = -1

Question 2

(a) M_1 = ((0, 1), (1, 0)) **B1**

(a) M_2 = ((1, 0), (0, 3)) **B1**

(a) Answer: M_1 = ((0, 1), (1, 0)), M_2 = ((1, 0), (0, 3))

(b) multiplies in the correct order: M_2 M_1 **M1**

(b) ((0, 1), (3, 0)) **A1**

(b) Answer: ((0, 1), (3, 0))

(c) multiplies the combined matrix by the position vector **M1**

(c) (-4, -6) **A1**

(c) Answer: (-4, -6)

(d) area scale factor = |det of the combined matrix| **M1**

(d) |-3| x 6 = 18 **A1**

(d) Answer: 18

Question 3

(a) attempts ad - bc, i.e. 4(2) - 7(1) **M1**

(a) det(M) = 1 cao **A1**

(a) Answer: det(M) = 1

(b) uses M^{-1} = (1/det(M)) [[d,-b],[-c,a]] with correct entries from M **M1**

(b) M^{-1} = [[2,-7],[-1,4]] cao **A1**

(b) Answer: M^{-1} = [[2,-7],[-1,4]]

(c) writes the system as $M(x,y)^T = (15,4)^T$ and multiplies both sides by M^{-1} **M1**

(c) x = 2 **A1**

(c) y = 1 **A1**

(c) Answer: x = 2, y = 1

Question 4

(a) expands the determinant along a row or column **M1**

(a) evaluates the 2 x 2 minors correctly **A1**

(a) det A = 6k + 0 (cso) **A1**

(a) Answer: det A = 6k + 0

(b) sets det A = 0 **M1**

(b) k = 0 **A1**

(b) Answer: k = 0

(c) det A = 0 **B1**

(c) singular, not invertible **B1**

(c) Answer: 0; not invertible

Question 5

(a) attempts the matrix product A x A **M1**

(a) at least two of the four entries correct **M1**

(a) all four entries correct, matching [[4+ab,5a],[5b,ab+9]] cso **A1**

(a) Answer: A^2 = [[4+ab,5a],[5b,ab+9]]

(b) equates 5a = 15 (from the (1,2) entries) **M1**

(b) a = 3 **A1**

(b) equates 5b = 5 (from the (2,1) entries) **M1**

(b) b = 1 **A1**

(b) verifies using the remaining entries, e.g. 4+ab = 4+3 = 7 and ab+9 = 3+9 = 12, both consistent with the given A^2 **B1**

(b) Answer: a = 3, b = 1

Question 6

(a) finds the image of the point (1,1) under Q, i.e. (2k+1, k+7) **M1**

(a) uses the condition that the two coordinates of the image must be equal, since the point lies on y = x **M1**

(a) correct equation 2k+1 = k+7 oe **A1**

(a) k = 6 cso **A1**

(a) Answer: k = 6

(b) substitutes k=6 to get Q = [[12,1],[3,10]] and finds the image of a general point (1,m), i.e. (12+m, 3+10m) **M1**

(b) correct image expression **A1**

(b) sets up the invariance condition (3+10m)/(12+m) = m **M1**

(b) correct quadratic m^2 + 2m - 3 = 0 oe **A1**

(b) solves the quadratic by a valid method, e.g. factorises as (m+3)(m-1) = 0 **M1**

(b) m = -3 (rejecting the already-known solution m=1), so the other invariant line is y = -3x **A1**

(b) Answer: y = -3x

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