1.13 Core Pure: Matrices and Transformations Depth: 2-hour Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The transformation T_1 is a shear with the y axis invariant, taking (1, 0) to (1, 3). The transformation T_2 is an enlargement with scale factor 3, centre the origin.

(a) Write down the 2 x 2 matrices M_1 and M_2 representing T_1 and T_2. **(2)**

(b) Find the matrix representing the transformation T_1 followed by T_2. **(2)**

(c) Find the image of the point (1, -2) under T_1 followed by T_2. **(2)**

(d) A triangle of area 6 is transformed by T_1 followed by T_2. Find the area of its image. **(2)**

(Total for Question 1 is 8 marks)

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2 The matrices A and B are given by A = ((5, 1), (-3, 2)) and B = ((-1, 2), (-3, 4)), where each matrix is written row by row.

(a) Find AB. **(2)**

(b) Find det A and, given that A is non-singular, find A^{-1}. **(4)**

(c) Use A^{-1} to solve the equations $5x + y = -14$ and $-3x + 2y = 11$. **(3)**

(Total for Question 2 is 9 marks)

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3 The matrix M is given by M = ((5, -1), (0, -2)).

(a) Find the characteristic equation of M and hence find its eigenvalues. **(3)**

(b) Find an eigenvector corresponding to each eigenvalue, giving each as a vector with integer components in simplest form. **(3)**

(c) Find M^4. **(2)**

(Total for Question 3 is 8 marks)

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4 The matrix M is given by M = ((-5, 2), (-2, 0)).

(a) Find the characteristic equation of M and hence find its eigenvalues. **(3)**

(b) Find an eigenvector corresponding to each eigenvalue, giving each as a vector with integer components in simplest form. **(3)**

(c) Find M^2. **(2)**

(Total for Question 4 is 8 marks)

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5 The matrix M is given by M = ((-3, -3), (-3, -3)).

(a) Find the characteristic equation of M and hence find its eigenvalues. **(3)**

(b) Find an eigenvector corresponding to each eigenvalue, giving each as a vector with integer components in simplest form. **(3)**

(c) Find M^4. **(2)**

(Total for Question 5 is 8 marks)

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6 The transformation T_1 is a reflection in the line y = x. The transformation T_2 is a shear with the x axis invariant, taking (0, 1) to (2, 1).

(a) Write down the 2 x 2 matrices M_1 and M_2 representing T_1 and T_2. **(2)**

(b) Find the matrix representing the transformation T_1 followed by T_2. **(2)**

(c) Find the image of the point (3, -1) under T_1 followed by T_2. **(2)**

(d) A triangle of area 6 is transformed by T_1 followed by T_2. Find the area of its image. **(2)**

(Total for Question 6 is 8 marks)

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7 The matrix C is defined by C = [[1,2,3],[0,4,5],[1,0,6]]. Find det(C), showing your method clearly.

(Total for Question 7 is 4 marks)

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8 The matrix A = [[2,0],[0,2]] represents an enlargement, centre O, scale factor 2. The matrix B = [[0,-1],[1,0]] represents a rotation of 90 degrees anticlockwise about O. The enlargement is applied first, followed by the rotation.

(a) Find the single matrix, BA, that represents the combined transformation. **(3)**

(b) Use your matrix from part (a) to find the image of the point (3,-1) under the combined transformation. **(2)**

(c) Describe the single geometrical transformation equivalent to the matrix BA. **(1)**

(Total for Question 8 is 6 marks)

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9 The matrix T is defined by T = [[3,-1],[2,4]]. Triangle OAB has vertices O(0,0), A(2,0) and B(0,3).

(a) Find det(T). **(2)**

(b) State the area of triangle OAB. **(1)**

(c) Find the area of the image of triangle OAB under the transformation represented by T. **(2)**

(d) The transformation represented by T is applied twice in succession. Find the area scale factor of this combined transformation. **(2)**

(Total for Question 9 is 7 marks)

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10 In three dimensions, a rotation of angle θ anticlockwise about the z-axis (viewed from the positive z-axis looking towards the origin) is represented by the matrix Rz(θ) = [[cos(θ),-sin(θ),0],[sin(θ),cos(θ),0],[0,0,1]].

(a) Write down Rz(90), the matrix representing a rotation of 90 degrees anticlockwise about the z-axis. **(2)**

(b) Find the image of the point (2,5,-3) under this rotation. **(2)**

(c) State the equation(s) of the invariant line of this transformation (the line consisting entirely of invariant points). **(2)**

(d) Find det(Rz(90)) and interpret its value geometrically. **(3)**

(Total for Question 10 is 9 marks)

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11 The matrix M(k) is defined by M(k) = [[k,1],[2,k]], where k is a real constant.

(a) Find det(M(k)) in terms of k. **(2)**

(b) Find the value(s) of k for which M(k) is singular. **(2)**

(c) When k = 3, find $M(3)^{-1}$. **(3)**

(d) Show that the only invariant point of the transformation represented by M(3) is the origin. **(3)**

(e) The transformation represented by M(3) is applied to the point (1,-2), followed by a translation by the vector (3,1). Find the coordinates of the final image point. **(2)**

(Total for Question 11 is 12 marks)

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12 A system of three equations in x, y and z is given by: x + 2y - z = 4; 2x - y + 3z = 3; 4x + 3y + z = k, where k is a constant.

(a) Show that the determinant of the coefficient matrix [[1,2,-1],[2,-1,3],[4,3,1]] is 0. **(3)**

(b) By finding a linear combination of the first two equations that produces the same left-hand side as the third equation, find the value of k for which the system is consistent. **(4)**

(c) For this value of k, find a vector equation of the line in which the three planes intersect. **(4)**

(Total for Question 12 is 11 marks)

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Mark scheme · 1.13 Core Pure: Matrices and Transformations Depth: 2-hour Pack

Question 1

(a) M_1 = ((1, 0), (3, 1)) **B1**

(a) M_2 = ((3, 0), (0, 3)) **B1**

(a) Answer: M_1 = ((1, 0), (3, 1)), M_2 = ((3, 0), (0, 3))

(b) multiplies in the correct order: M_2 M_1 **M1**

(b) ((3, 0), (9, 3)) **A1**

(b) Answer: ((3, 0), (9, 3))

(c) multiplies the combined matrix by the position vector **M1**

(c) (3, 3) **A1**

(c) Answer: (3, 3)

(d) area scale factor = |det of the combined matrix| **M1**

(d) |9| x 6 = 54 **A1**

(d) Answer: 54

Question 2

(a) multiplies rows of A by columns of B **M1**

(a) ((-8, 14), (-3, 2)) **A1**

(a) Answer: ((-8, 14), (-3, 2))

(b) det A = ad - bc **M1**

(b) det A = 13 **A1**

(b) uses the inverse formula (1/det)((d, -b), (-c, a)) **M1**

(b) A^{-1} = ((2/13, -1/13), (3/13, 5/13)) **A1**

(b) Answer: det A = 13, A^{-1} = ((2/13, -1/13), (3/13, 5/13))

(c) writes as a matrix equation A(x, y) = (b1, b2) and multiplies by A^{-1} **M1**

(c) x = -3 **A1**

(c) y = 1 **A1**

(c) Answer: x = -3, y = 1

Question 3

(a) forms det(M - λ I) = 0 **M1**

(a) λ^2 - 3lambda - 10 = 0 **A1**

(a) λ = -2 and λ = 5 **A1**

(a) Answer: λ = -2, 5

(b) solves (M - λ I) v = 0 for the first eigenvalue **M1**

(b) for λ = -2: (1, 7) **A1**

(b) for λ = 5: (1, 0) **A1**

(b) Answer: λ = -2: (1, 7); λ = 5: (1, 0)

(c) multiplies M by itself 4 times (or uses the diagonalisation PD^4P^{-1}) **M1**

(c) ((625, -87), (0, 16)) **A1**

(c) Answer: ((625, -87), (0, 16))

Question 4

(a) forms det(M - λ I) = 0 **M1**

(a) λ^2 + 5lambda + 4 = 0 **A1**

(a) λ = -4 and λ = -1 **A1**

(a) Answer: λ = -4, -1

(b) solves (M - λ I) v = 0 for the first eigenvalue **M1**

(b) for λ = -4: (2, 1) **A1**

(b) for λ = -1: (1, 2) **A1**

(b) Answer: λ = -4: (2, 1); λ = -1: (1, 2)

(c) multiplies M by itself 2 times (or uses the diagonalisation PD^2P^{-1}) **M1**

(c) ((21, -10), (10, -4)) **A1**

(c) Answer: ((21, -10), (10, -4))

Question 5

(a) forms det(M - λ I) = 0 **M1**

(a) λ^2 + 6lambda + 0 = 0 **A1**

(a) λ = -6 and λ = 0 **A1**

(a) Answer: λ = -6, 0

(b) solves (M - λ I) v = 0 for the first eigenvalue **M1**

(b) for λ = -6: (1, 1) **A1**

(b) for λ = 0: (1, -1) **A1**

(b) Answer: λ = -6: (1, 1); λ = 0: (1, -1)

(c) multiplies M by itself 4 times (or uses the diagonalisation PD^4P^{-1}) **M1**

(c) ((648, 648), (648, 648)) **A1**

(c) Answer: ((648, 648), (648, 648))

Question 6

(a) M_1 = ((0, 1), (1, 0)) **B1**

(a) M_2 = ((1, 2), (0, 1)) **B1**

(a) Answer: M_1 = ((0, 1), (1, 0)), M_2 = ((1, 2), (0, 1))

(b) multiplies in the correct order: M_2 M_1 **M1**

(b) ((2, 1), (1, 0)) **A1**

(b) Answer: ((2, 1), (1, 0))

(c) multiplies the combined matrix by the position vector **M1**

(c) (5, 3) **A1**

(c) Answer: (5, 3)

(d) area scale factor = |det of the combined matrix| **M1**

(d) |-1| x 6 = 6 **A1**

(d) Answer: 6

Question 7

expands the determinant along any row or column with correct cofactor structure **M1**

correct value for at least two of the three 2x2 minors used, e.g. 4x6-5x0=24, 0x6-5x1=-5, 0x0-4x1=-4 **A1**

combines the minors with the correct signs and coefficients **M1**

det(C) = 22 cao **A1**

Answer: det(C) = 22

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Question 8

(a) attempts the matrix product BA (rotation matrix multiplied by enlargement matrix, in the correct order) **M1**

(a) at least one row of BA correct **A1**

(a) BA = [[0,-2],[2,0]] cao **A1**

(a) Answer: BA = [[0,-2],[2,0]]

(b) multiplies BA by the column vector $(3,-1)^T$ **M1**

(b) image = (2,6) cao **A1**

(b) Answer: (2,6)

(c) describes BA as an enlargement, centre O, scale factor 2, combined with a rotation of 90 degrees anticlockwise about O; accept equivalent correct descriptions of this spiral similarity **B1**

(c) Answer: A rotation of 90 degrees anticlockwise about O combined with an enlargement, centre O, scale factor 2.

Question 9

(a) attempts 3(4) - (-1)(2) **M1**

(a) det(T) = 14 cao **A1**

(a) Answer: det(T) = 14

(b) area = 3 **B1**

(b) Answer: 3

(c) multiplies the area of OAB by |det(T)| **M1**

(c) area = 42 cao **A1**

(c) Answer: 42

(d) uses the area scale factor of T^2 equal to $(\det T)^2$ **M1**

(d) 196 cao **A1**

(d) Answer: 196

Question 10

(a) uses cos(90) = 0 and sin(90) = 1 **M1**

(a) Rz(90) = [[0,-1,0],[1,0,0],[0,0,1]] cao **A1**

(a) Answer: Rz(90) = [[0,-1,0],[1,0,0],[0,0,1]]

(b) multiplies Rz(90) by the column vector $(2,5,-3)^T$ **M1**

(b) (-5,2,-3) cao **A1**

(b) Answer: (-5,2,-3)

(c) identifies the invariant line as the z-axis **B1**

(c) gives its equation as x=0, y=0 (z takes any value) oe **B1**

(c) Answer: The z-axis: x = 0, y = 0 (z takes any value).

(d) attempts the 3x3 determinant expansion of Rz(90) **M1**

(d) det(Rz(90)) = 1 **A1**

(d) interprets: the transformation preserves volume (volume scale factor 1) and preserves orientation, consistent with Rz(90) being a rotation **B1**

(d) Answer: det(Rz(90)) = 1; volumes are preserved and orientation is preserved, as expected for a rotation.

Question 11

(a) attempts k(k) - 1(2) **M1**

(a) det(M(k)) = k^2 - 2 cao **A1**

(a) Answer: k^2 - 2

(b) sets k^2 - 2 = 0 **M1**

(b) $k = \sqrt{2}$ or $k = -\sqrt{2}$ (both required) **A1**

(b) Answer: $k = \sqrt{2}$ or $k = -\sqrt{2}$

(c) finds det(M(3)) = 9-2 = 7 **M1**

(c) uses M^{-1} = (1/det)[[d,-b],[-c,a]] with correct entries **M1**

(c) $M(3)^{-1}$ = (1/7)[[3,-1],[-2,3]] cao **A1**

(c) Answer: $M(3)^{-1}$ = (1/7)[[3,-1],[-2,3]]

(d) sets up $(M(3) - I)(x,y)^T = (0,0)^T$, i.e. the equations 2x+y=0 and 2x+2y=0 **M1**

(d) solves the equations simultaneously (e.g. subtracting to get y=0, hence x=0), or argues via det(M(3)-I) ≠ 0 **M1**

(d) concludes x=0, y=0 is the only solution, so the only invariant point is the origin cso **A1**

(d) Answer: The origin (0,0) is the only invariant point.

(e) finds $M(3)(1,-2)^T$ = (1,-4) and adds the translation vector (3,1) **M1**

(e) (4,-3) cao **A1**

(e) Answer: (4,-3)

Question 12

(a) attempts expansion of the determinant along any row or column with correct cofactor structure **M1**

(a) correct 2x2 minors, e.g. (-1x1-3x3)=-10, (2x1-3x4)=-10, (2x3-(-1)x4)=10 **A1**

(a) determinant = 0 cso **A1**

(a) Answer: det = 0

(b) attempts a combination such as 2 x (equation 1) + (equation 2) **M1**

(b) correct combined left-hand side 4x+3y+z, matching equation 3's left-hand side **A1**

(b) applies the same combination to the right-hand sides: 2(4)+3 **M1**

(b) k = 11 cao **A1**

(b) Answer: k = 11

(c) sets z=t and reduces the system to two equations in x and y: x+2y=4+t and 2x-y=3-3t **M1**

(c) solves these simultaneously for x and y in terms of t **M1**

(c) x = 2-t, y = 1+t (or equivalent correct expressions) **A1**

(c) vector equation r = (2,1,0) + t(-1,1,1) oe **A1**

(c) Answer: r = (2,1,0) + t(-1,1,1)

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