1.13 Core Pure: Matrices and Transformations Depth: Test Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The matrices A and B are defined by A = [[2,-1],[3,4]] and B = [[1,0],[-2,5]].

(a) Find the matrix AB. **(2)**

(b) Find the matrix BA. **(2)**

(c) State, with a reason, whether AB = BA. **(1)**

(Total for Question 1 is 5 marks)

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2 The matrices A and B are given by A = ((2, 2), (2, -4)) and B = ((-1, 3), (0, -2)), where each matrix is written row by row.

(a) Find AB. **(2)**

(b) Find det A and, given that A is non-singular, find A^{-1}. **(4)**

(c) Use A^{-1} to solve the equations 2x + 2y = 2 and 2x - 4y = 8. **(3)**

(Total for Question 2 is 9 marks)

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3 The matrix A is given by A = ((-1, 0, -3), (-3, 2, -1), (k, 2, -2)), written row by row, where k is a constant.

(a) Show that det A = 6k + 20. **(3)**

(b) Find the value of k for which A is singular. **(2)**

(c) State the value of det A when k = 4, and hence state whether A is invertible when k = 4. **(2)**

(Total for Question 3 is 7 marks)

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4 The matrix R represents a rotation about the origin O through 60 degrees anticlockwise, so that R = [[cos60,-sin60],[sin60,cos60]].

(a) Write R in exact (surd) form. **(2)**

(b) Find the exact image of the point (4,0) under R. **(2)**

(c) State the value of det(R) and interpret this value geometrically for the transformation R. **(2)**

(Total for Question 4 is 6 marks)

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5 The transformation T is represented by the matrix N = [[5,4],[1,2]].

(a) Find the equations of the two invariant lines of T that pass through the origin. **(6)**

(b) Determine which of the two invariant lines found in part (a) consists entirely of invariant points (that is, every point on the line maps to itself). **(2)**

(Total for Question 5 is 8 marks)

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6 The matrix P is defined by P = [[1,2,1],[0,1,-1],[2,3,0]].

(a) Show that det(P) = -3. **(3)**

(b) Find P^{-1}. **(5)**

(c) Hence solve the system of equations: x + 2y + z = 3, y - z = -6, 2x + 3y = 0. **(2)**

(Total for Question 6 is 10 marks)

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Mark scheme · 1.13 Core Pure: Matrices and Transformations Depth: Test Pack

Question 1

(a) attempts at least one correct row-by-column entry of AB **M1**

(a) AB = [[4,-5],[-5,20]] cao **A1**

(a) Answer: AB = [[4,-5],[-5,20]]

(b) attempts at least one correct row-by-column entry of BA **M1**

(b) BA = [[2,-1],[11,22]] cao **A1**

(b) Answer: BA = [[2,-1],[11,22]]

(c) states AB is not equal to BA, referencing that matrix multiplication is not commutative in general (e.g. comparing the (1,1) entries 4 and 2) **B1**

(c) Answer: AB does not equal BA; matrix multiplication is not commutative in general.

Question 2

(a) multiplies rows of A by columns of B **M1**

(a) ((-2, 2), (-2, 14)) **A1**

(a) Answer: ((-2, 2), (-2, 14))

(b) det A = ad - bc **M1**

(b) det A = -12 **A1**

(b) uses the inverse formula (1/det)((d, -b), (-c, a)) **M1**

(b) A^{-1} = ((1/3, 1/6), (1/6, -1/6)) **A1**

(b) Answer: det A = -12, A^{-1} = ((1/3, 1/6), (1/6, -1/6))

(c) writes as a matrix equation A(x, y) = (b1, b2) and multiplies by A^{-1} **M1**

(c) x = 2 **A1**

(c) y = -1 **A1**

(c) Answer: x = 2, y = -1

Question 3

(a) expands the determinant along a row or column **M1**

(a) evaluates the 2 x 2 minors correctly **A1**

(a) det A = 6k + 20 (cso) **A1**

(a) Answer: det A = 6k + 20

(b) sets det A = 0 **M1**

(b) k = -10/3 **A1**

(b) Answer: k = -10/3

(c) det A = 44 **B1**

(c) invertible **B1**

(c) Answer: 44; invertible

Question 4

(a) uses cos60 = 1/2 and sin60 = $\sqrt{3}/2$ **M1**

(a) R = [[1/2,-$\sqrt{3}$/2],[$\sqrt{3}$/2,1/2]] oe cao **A1**

(a) Answer: R = [[1/2,-$\sqrt{3}$/2],[$\sqrt{3}$/2,1/2]]

(b) multiplies R by the column vector $(4,0)^T$ **M1**

(b) image = (2, $2\sqrt{3}$) oe cao **A1**

(b) Answer: (2, $2\sqrt{3}$)

(c) det(R) = 1 **B1**

(c) correct interpretation: the transformation preserves area (area scale factor 1) and preserves orientation, consistent with R being a rotation **B1**

(c) Answer: det(R) = 1; areas are preserved (scale factor 1) and orientation is preserved, as expected for a rotation.

Question 5

(a) considers a general point (1,m) on a line y = mx through O and finds its image under N, i.e. (5+4m, 1+2m) **M1**

(a) sets up the invariance condition (1+2m)/(5+4m) = m, or equivalent **M1**

(a) correct quadratic $4m^2 + 3m - 1 = 0$ oe **A1**

(a) solves the quadratic by a valid method (factorising or the formula) **M1**

(a) m = 1/4 **A1**

(a) m = -1, giving invariant lines y = x/4 and y = -x **A1**

(a) Answer: y = x/4 and y = -x

(b) tests a general point (x,-x) on y=-x under N: $N(x,-x)^T$ = (5x-4x, x-2x) = (x,-x) **M1**

(b) concludes y = -x consists entirely of invariant points (every point maps to itself), unlike y = x/4, where points map to other points on the same line **A1**

(b) Answer: y = -x consists entirely of invariant points.

Question 6

(a) attempts expansion of det(P) along any row or column with correct cofactor structure **M1**

(a) correct 2x2 minors used, e.g. (1x0-(-1)x3)=3, (0x0-(-1)x2)=2, (0x3-1x2)=-2 **A1**

(a) det(P) = -3 cso **A1**

(a) Answer: det(P) = -3

(b) attempts at least four of the nine cofactors of P **M1**

(b) correct cofactor matrix [[3,-2,-2],[3,-2,1],[-3,1,1]] **A1**

(b) transposes the cofactor matrix to form the adjugate **M1**

(b) adjugate = [[3,3,-3],[-2,-2,1],[-2,1,1]] **A1**

(b) P^{-1} = (1/-3)[[3,3,-3],[-2,-2,1],[-2,1,1]] = [[-1,-1,1], [2/3,2/3,-1/3],[2/3,-1/3,-1/3]] cao (unsimplified equivalent forms accepted) **A1**

(b) Answer: P^{-1} = [[-1,-1,1],[2/3,2/3,-1/3],[2/3,-1/3,-1/3]]

(c) multiplies P^{-1} by the column vector $(3,-6,0)^T$ **M1**

(c) x = 3, y = -2, z = 4 cao **A1**

(c) Answer: x = 3, y = -2, z = 4

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