1.14 Core Pure: Further Calculus and Series Depth: 1-hour Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = e^{3x}/(1 + x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 1 is 6 marks)

---

2 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(6r^2 + 3r + 0)$ can be written as $n(12n^2 + 27n + 15)/6$. **(4)**

(b) Hence evaluate the sum from r = 2 to r = 10 of $(6r^2 + 3r + 0)$. **(4)**

(Total for Question 2 is 8 marks)

---

3 This question is about improper integrals.

(a) Show that the improper integral of 6/x from x = 1 to infinity does not converge. **(3)**

(Total for Question 3 is 3 marks)

© Revision Library | revisionlibrary.co.uk

4 This question is about the series with r-th term $u_r = 6/(r(r + 3))$.

(a) Show that $6/(r(r + 3)) = (2)(1/r - 1/(r + 3))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 20. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 4 is 8 marks)

---

5 $f(x) = e^x \ln(1 + 2x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 5 is 6 marks)

© Revision Library | revisionlibrary.co.uk

6 The curve C has equation $y = \ln x$, for $x \geq 1$. The region R is bounded by C, the y-axis, and the lines y=0 and y=1. R is rotated through $2*\pi$ radians about the y-axis to form a solid of revolution.

(a) Show that $x^2 = e^{2y}$. **(2)**

(b) Find the exact volume of the solid formed, giving your answer in terms of e. **(5)**

(c) Give the volume found in part (b) correct to 3 significant figures. **(1)**

(Total for Question 6 is 8 marks)

---

7 The curve C has equation $y = 1/x$, for $x \geq 1$. The region R is bounded by C, the x-axis, and the line x=1, and extends without bound as x -> infinity. R is rotated through $2*\pi$ radians about the x-axis to form an infinite solid of revolution S.

(a) Show that the volume of S formed between x=1 and x=b, where b>1, is $V(b) = \pi(1 - 1/b)$. **(4)**

(b) Hence show that, as b -> infinity, the volume of S converges to a finite limit, and state this limit. **(3)**

(c) It is a standard result, which you may assume without proof, that the corresponding surface area of S between x=1 and x=b is $A(b) = 2*\pi *$ integral from x=1 to x=b of $(1/x)*\sqrt{1+1/x^4}$ dx, and that A(b) -> infinity as b -> infinity. Using your results from parts (a) and (b), comment on what this implies about the volume and surface area of the infinite solid S. **(2)**

(d) A second, finite, solid is formed by rotating C about the x-axis between x=1 and x=10 only. Find its exact volume, giving your answer in the form $k*\pi$, where k is a rational number to be found. **(3)**

(Total for Question 7 is 12 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.14 Core Pure: Further Calculus and Series Depth: 1-hour Pack

Question 1

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 + 2x + (5/2)x^2 + 2x^3$ **A1**

(a) Answer: $1 + 2x + (5/2)x^2 + 2x^3$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 1.22700 **A1**

(b) Answer: 1.22700

Question 2

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $6n(n + 1)(2n + 1)/6 + 3n(n + 1)/2$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(12n^2 + 27n + 15)/6$ (cso) **A1**

(a) Answer: $n(12n^2 + 27n + 15)/6$

(b) S(10) - S(1) **M1**

(b) S(10) = 2475 **A1**

(b) S(1) = 9 **A1**

(b) 2466 **A1**

(b) Answer: 2466

Question 3

(a) integrates: 6 ln x **M1**

(a) evaluates between 1 and t **M1**

(a) 6 ln t tends to infinity as t tends to infinity, so the integral diverges **A1**

(a) Answer: diverges

Question 4

(a) writes $1/r - 1/(r + 3)$ over a common denominator **M1**

(a) $= 3/(r(r + 3))$ and multiplies by 2 (cso) **A1**

(a) Answer: $6/(r(r + 3)) = (2)(1/r - 1/(r + 3))$

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: $(1 + 1/2 + 1/3) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3))$ **A1**

(b) $S_n = (2)[11/6 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3)]$ **A1**

(b) $S_{20} = 6010/1771$ **A1**

(b) Answer: $S_{20} = 6010/1771$

(c) the terms in n tend to 0 **M1**

(c) 11/3 **A1**

(c) Answer: 11/3

Question 5

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $2x + (5/3)x^3 - 2x^4 + (209/60)x^5$ **A1**

(a) Answer: $2x + (5/3)x^3 - 2x^4 + (209/60)x^5$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.20150 **A1**

(b) Answer: 0.20150

Question 6

(a) rearranges $y = \ln x$ to $x = e^y$ **M1**

(a) squares to obtain $x^2 = e^{2y}$ cso **A1**

(a) Answer: $x^2 = e^{2y}$ (shown)

(b) V = π * integral from y=0 to y=1 of x^2 dy soi **B1**

(b) substitutes $x^2 = e^{2y}$ **M1**

(b) integrates e^{2y} to $(1/2)e^{2y}$ **M1**

(b) substitutes the limits y=1 and y=0 correctly **M1**

(b) $V = (e^2 - 1)*\pi/2$ oe cao **A1**

(b) Answer: $V = (e^2 - 1)\pi/2$

(c) V = 10.0 awrt (3 s.f.) **B1**

(c) Answer: 10.0 (awrt)

Question 7

(a) uses $V(b) = \pi$ * integral from x=1 to x=b of y^2 dx with $y^2 = 1/x^2$ **M1**

(a) integrates x^{-2} to -1/x **M1**

(a) substitutes the limits b and 1 correctly **M1**

(a) $V(b) = \pi(1 - 1/b)$ cso **A1**

(a) Answer: $V(b) = \pi(1 - 1/b)$ (shown)

(b) considers the limit as b -> infinity of 1/b **M1**

(b) 1/b -> 0 as b -> infinity **A1**

(b) V(b) -> π, so the volume converges to π cao **A1**

(b) Answer: The volume converges to π

(c) recognises that S has a finite volume (π, from part (b)) but an infinite surface area (since A(b) -> infinity) **B1**

(c) gives a correct comment, e.g. that a solid can enclose a finite volume even though its bounding surface has infinite area, so finite volume does not imply finite surface area **B1**

(c) Answer: S has finite volume π but infinite surface area; a solid can have a finite volume while its surface area is unbounded.

(d) uses V = π * integral from x=1 to x=10 of $(1/x)^2$ dx **M1**

(d) integrates and substitutes the limits 10 and 1 correctly **M1**

(d) $V = 9*\pi/10$ cao **A1**

(d) Answer: V = 9pi/10

© Revision Library | revisionlibrary.co.uk