1.14 Core Pure: Further Calculus and Series Depth: 30-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to 1 of $1/(x^2 + 1)$ dx. **(4)**

(Total for Question 1 is 4 marks)

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2 This question is about the series with r-th term $u_r = 6/(r(r + 3))$.

(a) Show that $6/(r(r + 3)) = (2)(1/r - 1/(r + 3))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 18. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 2 is 8 marks)

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3 The curve C has equation $y = e^{-x/2}$, for $0 \leq x \leq \ln 4$. The region R is bounded by C, the x-axis, and the lines x=0 and x=ln4. R is rotated through 2*π radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid formed is V = π * integral from x=0 to x=ln4 of e^{-x} dx. **(2)**

(b) Hence find the exact value of V, giving your answer in the form k*π, where k is a rational number to be found. **(4)**

(Total for Question 3 is 6 marks)

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4 f(x) = cos x.

(a) Write down the Maclaurin series for f(x), up to and including the term in x^4, and state the range of values of x for which it is valid. **(3)**

(b) By substituting x=0.5 into your series from part (a), find an approximation for cos(0.5), giving your answer to 4 decimal places. **(2)**

(c) Given that cos(0.5) = 0.87758, correct to 5 decimal places, calculate the percentage error in the approximation found in part (b), giving your answer to 2 significant figures. **(2)**

(Total for Question 4 is 7 marks)

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Mark scheme · 1.14 Core Pure: Further Calculus and Series Depth: 30-minute Pack

Question 1

(a) recognises the standard form $1/(x^2 + a^2)$ with a = 1 **M1**

(a) integrates: (1) arctan(x) **M1**

(a) (1) arctan(1) = (1)(π/4) **A1**

(a) π/4 **A1**

(a) Answer: π/4

Question 2

(a) writes 1/r - 1/(r + 3) over a common denominator **M1**

(a) = 3/(r(r + 3)) and multiplies by 2 (cso) **A1**

(a) Answer: 6/(r(r + 3)) = (2)(1/r - 1/(r + 3))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3)) **A1**

(b) S_n = (2)[11/6 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3)] **A1**

(b) S_{18} = 4477/1330 **A1**

(b) Answer: S_{18} = 4477/1330

(c) the terms in n tend to 0 **M1**

(c) 11/3 **A1**

(c) Answer: 11/3

Question 3

(a) uses V = π * integral y^2 dx with the correct limits 0 and ln4 **B1**

(a) $y^2 = (e^{-x/2})^2 = e^{-x}$, giving V = π * integral from 0 to ln4 of e^{-x} dx cso **A1**

(a) Answer: V = π * integral from x=0 to x=ln4 of e^{-x} dx (shown)

(b) integrates e^{-x} to $-e^{-x}$ **M1**

(b) substitutes the limits ln4 and 0 correctly **M1**

(b) uses e^{-l}n4 = 1/4 to obtain integral = 3/4 oe **A1**

(b) V = 3*π/4 cao **A1**

(b) Answer: V = 3pi/4

Question 4

(a) cos x = 1 - x^2/2 oe **B1**

(a) + x^4/24 oe **B1**

(a) valid for all real x **B1**

(a) Answer: cos x = 1 - x^2/2 + x^4/24 + ...; valid for all real x

(b) substitutes x=0.5 into 1 - x^2/2 + x^4/24 **M1**

(b) = 0.8776 (4 d.p.) cao **A1**

(b) Answer: 0.8776

(c) calculates |approximation - true value| / true value **M1**

(c) = 0.0028% awrt (2 s.f.) cao (allow 0.0023% if the 4 d.p. value 0.8776 from part (b) is used) **A1**

(c) Answer: 0.0028%

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