1.14 Core Pure: Further Calculus and Series Depth: Test Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The improper integral I = integral from x=1 to x=infinity of $4/x^3$ dx is defined as I = $\lim_{b \to \infty}$ integral from x=1 to x=b of $4/x^3$ dx.

(a) Show that integral from x=1 to x=b of $4/x^3$ dx = $2 - 2/b^2$. **(3)**

(b) Hence show that I converges, and find its exact value. **(2)**

(Total for Question 1 is 5 marks)

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2 The function $f(x) = 1/\sqrt{x-2}$ is undefined at x=2. The improper integral J = integral from x=2 to x=6 of $1/\sqrt{x-2}$ dx is defined as J = $\lim_{a \to 2+}$ integral from x=a to x=6 of $1/\sqrt{x-2}$ dx.

(a) Find integral from x=a to x=6 of $1/\sqrt{x-2}$ dx, in terms of a. **(3)**

(b) Determine whether J converges, giving its exact value if it does. **(3)**

(Total for Question 2 is 6 marks)

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3 This question is about the series with r-th term $u_r = 6/(r(r + 3))$.

(a) Show that $6/(r(r + 3)) = (2)(1/r - 1/(r + 3))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 19. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 3 is 8 marks)

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4 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to $\sqrt{3}$ of $1/(x^2 + 9)$ dx. **(4)**

(Total for Question 4 is 4 marks)

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5 A chemical process cools a sample so that its temperature, θ degrees Celsius, t minutes after the process begins, satisfies $\theta = 20 + 60*e^{-0.2t}$ for $t \geq 0$.

(a) Find the mean temperature of the sample over the first 10 minutes, giving your exact value and also your answer correct to 3 significant figures. **(5)**

(b) State one limitation of using this mean value to estimate the sample's typical temperature during the cooling process. **(1)**

(Total for Question 5 is 6 marks)

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6 You may use, without proof, the standard series $e^x = 1 + x + x^2/2! + x^3/3! + ...$ and $\cos x = 1 - x^2/2! + x^4/4! - ...$

(a) Write down the Maclaurin expansions of e^x and cos x, up to and including the term in x^3. **(2)**

(b) Hence show that $e^x * \cos x = 1 + x - (1/3)x^3 + O(x^4)$. **(6)**

(Total for Question 6 is 8 marks)

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7 Given that $1/(r(r+2)) = (1/2)(1/r - 1/(r+2))$ for r a positive integer,

(a) Verify that this identity is correct by combining the right-hand side into a single fraction. **(2)**

(b) Using the method of differences, show that $\sum_{r=1}^{n} 1/(r(r+2)) = n(3n+5)/(4(n+1)(n+2))$. **(6)**

(c) Hence find the exact value of $\sum_{r=1}^{20} 1/(r(r+2))$. **(2)**

(Total for Question 7 is 10 marks)

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Mark scheme · 1.14 Core Pure: Further Calculus and Series Depth: Test Pack

Question 1

(a) integrates $4x^{-3}$ to $-2x^{-2}$ (oe) **M1**

(a) substitutes the limits b and 1 correctly into $-2x^{-2}$ **M1**

(a) = $2 - 2/b^2$ cso **A1**

(a) Answer: $2 - 2/b^2$ (shown)

(b) as b -> infinity, $2/b^2$ -> 0 **B1**

(b) I = 2 (converges) cao **A1**

(b) Answer: I = 2

Question 2

(a) integrates $(x-2)^{-1/2}$ to $2(x-2)^{1/2}$ (oe) **M1**

(a) substitutes the limits 6 and a correctly **M1**

(a) = $4 - 2\sqrt{a-2}$ oe cao **A1**

(a) Answer: $4 - 2\sqrt{a-2}$

(b) considers the limit as a -> 2+ of $2\sqrt{a-2}$ **M1**

(b) $2\sqrt{a-2}$ -> 0 as a -> 2+ **A1**

(b) J = 4 (converges) cao **A1**

(b) Answer: J = 4

Question 3

(a) writes 1/r - 1/(r + 3) over a common denominator **M1**

(a) = 3/(r(r + 3)) and multiplies by 2 (cso) **A1**

(a) Answer: 6/(r(r + 3)) = (2)(1/r - 1/(r + 3))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3)) **A1**

(b) S_n = (2)[11/6 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3)] **A1**

(b) S_{19} = 2603/770 **A1**

(b) Answer: S_{19} = 2603/770

(c) the terms in n tend to 0 **M1**

(c) 11/3 **A1**

(c) Answer: 11/3

Question 4

(a) recognises the standard form $1/(x^2 + a^2)$ with a = 3 **M1**

(a) integrates: (1/3) arctan(x/3) **M1**

(a) (1/3) arctan($1/\sqrt{3}$) = (1/3)(π/6) **A1**

(a) π/18 **A1**

(a) Answer: π/18

Question 5

(a) integrates the constant term 20 to 20t **M1**

(a) integrates $60*e^{-0.2t}$ to $-300*e^{-0.2t}$ oe **M1**

(a) substitutes the limits 10 and 0, and divides by 10 (the interval length) **M1**

(a) exact mean = $50 - 30*e^{-2}$ oe cao **A1**

(a) mean = 45.9 awrt (3 s.f.) **A1**

(a) Answer: Mean = $50 - 30e^{-2}$, awrt 45.9

(b) gives a sensible limitation, e.g. the model $\theta = 20+60e^{-0.2}t$ may not hold exactly in practice (real cooling can be affected by draughts, insulation, or measurement error), so the calculated mean is only as reliable as the model **B1**

(b) Answer: Any sensible limitation, e.g. the exponential cooling model may not describe the real sample exactly, so the mean value is only an estimate based on the model.

Question 6

(a) $e^x = 1 + x + x^2/2 + x^3/6$ oe **B1**

(a) cos x = $1 - x^2/2$ (+ $0x^3$) oe **B1**

(a) Answer: $e^x = 1+x+x^2/2+x^3/6$; cos x = $1 - x^2/2$

(b) multiplies the two series together **M1**

(b) identifies the constant term as 1x1=1 **M1**

(b) identifies the x^1 coefficient as 1x1 + 1x0 = 1 **M1**

(b) identifies the x^2 coefficient as 1x(-1/2)+1x0+(1/2)x1 = 0 **M1**

(b) identifies the x^3 coefficient as 1x0+1x(-1/2)+(1/2)x0+(1/6)x1 = -1/2+1/6 = -1/3 **A1**

(b) e^x*cos x = $1 + x - (1/3)x^3$ + ... cso **A1**

(b) Answer: e^x cos x = $1 + x - x^3/3$ + ...

Question 7

(a) combines (1/2)(1/r - 1/(r+2)) over the common denominator r(r+2) **M1**

(a) simplifies to 1/(r(r+2)), matching the left-hand side cso **A1**

(a) Answer: Identity verified

(b) writes $\sum_{r=1}^{n} 1/(r(r+2)) = (1/2) \sum_{r=1}^{n} (1/r - 1/(r+2))$ **M1**

(b) lists sufficient terms of the sum (e.g. r=1,2,...,n-1,n) to show the cancellation pattern **M1**

(b) identifies the surviving terms as 1 + 1/2 - 1/(n+1) - 1/(n+2) **A1**

(b) combines these four terms over a common denominator **M1**

(b) simplifies the numerator to $3n^2+5n$ **A1**

(b) sum = n(3n+5)/(4(n+1)(n+2)) cso **A1**

(b) Answer: $\sum_{r=1}^{n} 1/(r(r+2)) = n(3n+5)/(4(n+1)(n+2))$

(c) substitutes n=20 into the formula from part (b) **M1**

(c) = 325/462 cao **A1**

(c) Answer: 325/462

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