1.14 Core Pure: Further Calculus and Series Depth

EDEXCEL 9FM0 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question is about the series with r-th term $u_r = 5/(r(r + 3))$.

(a) Show that $5/(r(r + 3)) = (5/3)(1/r - 1/(r + 3))$. **(2)**

(b) Use the method of differences to find the sum from r = 1 to n of u_r, and evaluate the sum from r = 1 to 11. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 1 is 8 marks)

2 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to 1 of $1/(x^2 + 1)\,dx$. **(4)**

(Total for Question 2 is 4 marks)

3 This question is about the series with r-th term $u_r = 6/(r(r + 3))$.

(a) Show that $6/(r(r + 3)) = (2)(1/r - 1/(r + 3))$. **(2)**

(b) Use the method of differences to find the sum from $r = 1$ to n of u_r, and evaluate the sum from $r = 1$ to 18. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 3 is 8 marks)

4 $f(x) = e^{3x}/(1 + x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 4 is 6 marks)

5 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(6r^2 + 3r + 0)$ can be written as $n(12n^2 + 27n + 15)/6$. **(4)**

(b) Hence evaluate the sum from r = 2 to r = 10 of $(6r^2 + 3r + 0)$. **(4)**

(Total for Question 5 is 8 marks)

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6 This question is about improper integrals.

(a) Show that the improper integral of 6/x from x = 1 to infinity does not converge. **(3)**

(Total for Question 6 is 3 marks)

7 This question is about the series with r-th term $u_r = 6/(r(r + 3))$.

(a) Show that $6/(r(r + 3)) = (2)(1/r - 1/(r + 3))$. **(2)**

(b) Use the method of differences to find the sum from $r = 1$ to n of u_r, and evaluate the sum from $r = 1$ to 20. **(4)**

(c) Hence find the sum to infinity of the series. **(2)**

(Total for Question 7 is 8 marks)

8 $f(x) = e^x \ln(1 + 2x)$.

(a) Use the standard Maclaurin series to find the series expansion of f(x) in ascending powers of x, up to and including the first four non-zero terms. **(4)**

(b) Hence find an approximation for f(0.1), to 5 decimal places, using these four terms. **(2)**

(Total for Question 8 is 6 marks)

9 Use the standard results for the sums of r and r^2 to answer this question.

(a) Show that the sum from r = 1 to n of $(6r^2 - 3r + 8)$ can be written as $n(12n^2 + 9n + 45)/6$. **(4)**

(b) Hence evaluate the sum from r = 5 to r = 17 of $(6r^2 - 3r + 8)$. **(4)**

(Total for Question 9 is 8 marks)

10 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to $\sqrt{2}/2$ of $1/\sqrt{1 - x^2}$ dx. **(4)**

(Total for Question 10 is 4 marks)

11 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = x^2 + 3x$ over the interval $2 \leq x \leq 5$. **(4)**

(Total for Question 11 is 4 marks)

12 The region R is bounded by the curve $y = \sin(x)$, for $0 \leq x \leq \pi$, and the x axis. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 12 is 4 marks)

13 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = \sin(x)$ over the interval $0 \leq x \leq \pi/1$. **(3)**

(Total for Question 13 is 3 marks)

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14 The curve C has equation $y = e^{-x/2}$, for $0 \leq x \leq \ln 4$. The region R is bounded by C, the x-axis, and the lines x=0 and x=ln4. R is rotated through $2*\pi$ radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid formed is V = π * integral from x=0 to x=ln4 of e^{-x} dx. **(2)**

(b) Hence find the exact value of V, giving your answer in the form $k*\pi$, where k is a rational number to be found. **(4)**

(Total for Question 14 is 6 marks)

15 The region R is bounded by the curve $y = x^2 + 4$, the coordinate axes and the line $x = 1$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 15 is 4 marks)

16 This question is about improper integrals.

(a) Show that the improper integral of $4/x^2$ from $x = 1$ to infinity converges, and find its value. **(4)**

(Total for Question 16 is 4 marks)

17 This question is about improper integrals.

(a) Find the value of the integral from 0 to infinity of $5e^{-5x}$ dx. **(3)**

(Total for Question 17 is 3 marks)

18 The region R is bounded by the curve $y = e^{3x}$, the coordinate axes and the line $x = 3$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(3)**

(Total for Question 18 is 3 marks)

19 The region R is bounded by the curve $y = \sqrt{x}$, the x axis and the line $x = 4$. The region R is rotated through 2pi radians about the x axis.

(a) Write down an integral for the volume of the solid formed, and evaluate it exactly. **(4)**

(Total for Question 19 is 4 marks)

20 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = 4/x$ over the interval $2 \leq x \leq 6$, in exact form. **(3)**

(Total for Question 20 is 3 marks)

21 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = 3/x$ over the interval $1 \leq x \leq 5$, in exact form. **(3)**

(Total for Question 21 is 3 marks)

22 Use the method of differences, or standard results, to answer this question.

(a) Show that the sum from $r = 1$ to n of $6r(r + 1)(r + 2) = 6n(n + 1)(n + 2)(n + 3)/4$. **(3)**

(b) Hence evaluate the sum from $r = 2$ to 15 of $6r(r + 1)(r + 2)$. **(2)**

(Total for Question 22 is 5 marks)

23 The mean value of a function f over the interval [a,b] is defined as (1/(b-a)) * integral from x=a to x=b of f(x) dx. Find the mean value of $f(x) = 3*\sin^2(x)$ over the interval $[0, \pi]$, using the identity $\sin^2(x) = (1-\cos(2x))/2$.

(Total for Question 23 is 5 marks)

24 $f(x) = \cos x$.

(a) Write down the Maclaurin series for f(x), up to and including the term in x^4, and state the range of values of x for which it is valid. **(3)**

(b) By substituting x=0.5 into your series from part (a), find an approximation for cos(0.5), giving your answer to 4 decimal places. **(2)**

(c) Given that cos(0.5) = 0.87758, correct to 5 decimal places, calculate the percentage error in the approximation found in part (b), giving your answer to 2 significant figures. **(2)**

(Total for Question 24 is 7 marks)

25 The curve C has equation $y = \ln x$, for $x \geq 1$. The region R is bounded by C, the y-axis, and the lines y=0 and y=1. R is rotated through 2*π radians about the y-axis to form a solid of revolution.

(a) Show that $x^2 = e^{2y}$. **(2)**

(b) Find the exact volume of the solid formed, giving your answer in terms of e. **(5)**

(c) Give the volume found in part (b) correct to 3 significant figures. **(1)**

(Total for Question 25 is 8 marks)

26 $f(x) = \ln(1+2x)$.

(a) Show that $f'(x) = 2/(1+2x)$, and find $f''(x)$ and $f'''(x)$. **(3)**

(b) Hence find the Maclaurin series for $f(x)$, up to and including the term in x^3. **(3)**

(c) State the range of values of x for which this expansion is valid. **(1)**

(Total for Question 26 is 7 marks)

27 Let $f(r) = r(r+1)(r+2)(r+3)$.

(a) Show that $f(r) - f(r-1) = 4r(r+1)(r+2)$. **(3)**

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} r(r+1)(r+2) = (1/4)n(n+1)(n+2)(n+3)$. **(5)**

(c) Hence find $\sum_{r=1}^{15} r(r+1)(r+2)$. **(2)**

(Total for Question 27 is 10 marks)

28 Using the standard results $\sum_{r=1}^{n} r = n(n+1)/2$, $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$ and $\sum_{r=1}^{n} r^3 = (n(n+1)/2)^2$,

(a) Show that $\sum_{r=1}^{n} r(2r-1)(r+3) = (1/6)n(n+1)(3n^2+13n-4)$. **(6)**

(b) Hence find $\sum_{r=1}^{12} r(2r-1)(r+3)$. **(2)**

(Total for Question 28 is 8 marks)

29 The curve C has equation $y = 1/x$, for $x \geq 1$. The region R is bounded by C, the x-axis, and the line x=1, and extends without bound as x -> infinity. R is rotated through $2*\pi$ radians about the x-axis to form an infinite solid of revolution S.

(a) Show that the volume of S formed between x=1 and x=b, where b>1, is $V(b) = \pi(1 - 1/b)$. **(4)**

(b) Hence show that, as b -> infinity, the volume of S converges to a finite limit, and state this limit. **(3)**

(c) It is a standard result, which you may assume without proof, that the corresponding surface area of S between x=1 and x=b is $A(b) = 2*\pi *$ integral from x=1 to x=b of $(1/x)*\sqrt{1+1/x^4}$ dx, and that A(b) -> infinity as b -> infinity. Using your results from parts (a) and (b), comment on what this implies about the volume and surface area of the infinite solid S. **(2)**

(d) A second, finite, solid is formed by rotating C about the x-axis between x=1 and x=10 only. Find its exact volume, giving your answer in the form k*π, where k is a rational number to be found. **(3)**

(Total for Question 29 is 12 marks)

30 Given the identity 2*sin(1/2)*cos(r) = sin(r+1/2) - sin(r-1/2), which holds for all real r,

(a) Show that $\sum_{r=1}^{n} \cos(r) = (\sin(n+1/2) - \sin(1/2)) / (2*\sin(1/2))$. **(6)**

(b) Hence find, correct to 3 significant figures, the value of $\sum_{r=1}^{50} \cos(r)$, where r is measured in radians. **(3)**

(Total for Question 30 is 9 marks)

Mark scheme · 1.14 Core Pure: Further Calculus and Series Depth

Question 1

(a) writes 1/r - 1/(r + 3) over a common denominator **M1**

(a) = 3/(r(r + 3)) and multiplies by 5/3 (cso) **A1**

(a) Answer: 5/(r(r + 3)) = (5/3)(1/r - 1/(r + 3))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3)) **A1**

(b) S_n = (5/3)[11/6 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3)] **A1**

(b) S_{11} = 2915/1092 **A1**

(b) Answer: S_{11} = 2915/1092

(c) the terms in n tend to 0 **M1**

(c) 55/18 **A1**

(c) Answer: 55/18

Question 2

(a) recognises the standard form $1/(x^2 + a^2)$ with a = 1 **M1**

(a) integrates: (1) arctan(x) **M1**

(a) (1) arctan(1) = (1)(π/4) **A1**

(a) π/4 **A1**

(a) Answer: π/4

Question 3

(a) writes 1/r - 1/(r + 3) over a common denominator **M1**

(a) = 3/(r(r + 3)) and multiplies by 2 (cso) **A1**

(a) Answer: 6/(r(r + 3)) = (2)(1/r - 1/(r + 3))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3)) **A1**

(b) S_n = (2)[11/6 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3)] **A1**

(b) S_{18} = 4477/1330 **A1**

(b) Answer: S_{18} = 4477/1330

(c) the terms in n tend to 0 **M1**

(c) 11/3 **A1**

(c) Answer: 11/3

Question 4

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $1 + 2x + (5/2)x^2 + 2x^3$ **A1**

(a) Answer: $1 + 2x + (5/2)x^2 + 2x^3$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 1.22700 **A1**

(b) Answer: 1.22700

Question 5

(a) uses n(n + 1)(2n + 1)/6 for the r^2 term and n(n + 1)/2 for the r term **M1**

(a) writes 6n(n + 1)(2n + 1)/6 + 3n(n + 1)/2 over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(12n^2 + 27n + 15)/6$ (cso) **A1**

(a) Answer: $n(12n^2 + 27n + 15)/6$

(b) S(10) - S(1) **M1**

(b) S(10) = 2475 **A1**

(b) S(1) = 9 **A1**

(b) 2466 **A1**

(b) Answer: 2466

Question 6

(a) integrates: 6 ln x **M1**

(a) evaluates between 1 and t **M1**

(a) 6 ln t tends to infinity as t tends to infinity, so the integral diverges **A1**

(a) Answer: diverges

Question 7

(a) writes 1/r - 1/(r + 3) over a common denominator **M1**

(a) = 3/(r(r + 3)) and multiplies by 2 (cso) **A1**

(a) Answer: 6/(r(r + 3)) = (2)(1/r - 1/(r + 3))

(b) writes out the first few and last few terms and cancels **M1**

(b) the remaining terms: (1 + 1/2 + 1/3) - (1/(n + 1) + 1/(n + 2) + 1/(n + 3)) **A1**

(b) S_n = (2)[11/6 - 1/(n + 1) - 1/(n + 2) - 1/(n + 3)] **A1**

(b) S_{20} = 6010/1771 **A1**

(b) Answer: S_{20} = 6010/1771

(c) the terms in n tend to 0 **M1**

(c) 11/3 **A1**

(c) Answer: 11/3

Question 8

(a) writes down the standard series for each factor **M1**

(a) multiplies the series, collecting terms up to the required order **M1**

(a) any two correct terms **A1**

(a) $2x + (5/3)x^3 - 2x^4 + (209/60)x^5$ **A1**

(a) Answer: $2x + (5/3)x^3 - 2x^4 + (209/60)x^5$

(b) substitutes x = 0.1 into the expansion **M1**

(b) 0.20150 **A1**

(b) Answer: 0.20150

Question 9

(a) uses $n(n + 1)(2n + 1)/6$ for the r^2 term and $n(n + 1)/2$ for the r term **M1**

(a) writes $6n(n + 1)(2n + 1)/6 - 3n(n + 1)/2 + 8n$ over a common denominator **M1**

(a) expands and collects like terms **A1**

(a) $n(12n^2 + 9n + 45)/6$ (cso) **A1**

(a) Answer: $n(12n^2 + 9n + 45)/6$

(b) S(17) - S(4) **M1**

(b) S(17) = 10387 **A1**

(b) S(4) = 182 **A1**

(b) 10205 **A1**

(b) Answer: 10205

Question 10

(a) recognises the standard form $1/\sqrt{a^2 - x^2}$ with a = 1 **M1**

(a) integrates: arcsin(x) **M1**

(a) $\arcsin(\sqrt{2}/2) - \arcsin(0)$ **A1**

(a) $\pi/4$ **A1**

(a) Answer: $\pi/4$

Question 11

(a) mean value = (1/(5 - 2)) integral of f(x) dx **M1**

(a) integrates: $1/3x^3 + 3/2x^2$ **M1**

(a) integral = 141/2 **A1**

(a) mean value = 47/2 **A1**

(a) Answer: 47/2

Question 12

(a) V = π integral of y^2 dx **M1**

(a) uses $\sin^2(x) = (1 - \cos(2x))/2$ **M1**

(a) integrates and substitutes the limits 0 and π **M1**

(a) $V = \pi^2/2$ **A1**

(a) Answer: $\pi^2/2$

Question 13

(a) mean value = (1/(π)) integral of sin(x) dx **M1**

(a) integral = [-cos(x)] from 0 to π = 2 **M1**

(a) mean value = 2/π **A1**

(a) Answer: 2/π

Question 14

(a) uses V = π * integral y^2 dx with the correct limits 0 and ln4 **B1**

(a) $y^2 = (e^{-x/2})^2 = e^{-x}$, giving V = π * integral from 0 to ln4 of e^{-x} dx cso **A1**

(a) Answer: V = π * integral from x=0 to x=ln4 of e^{-x} dx (shown)

(b) integrates e^{-x} to $-e^{-x}$ **M1**

(b) substitutes the limits ln4 and 0 correctly **M1**

(b) uses $e^{-l}n4 = 1/4$ to obtain integral = 3/4 oe **A1**

(b) V = 3*π/4 cao **A1**

(b) Answer: V = 3pi/4

Question 15

(a) V = π integral of $(x^2 + 4)^2$ dx **M1**

(a) expands: $x^4 + 8x^2 + 16$ **M1**

(a) integrates term by term and substitutes the limits **M1**

(a) V = 283pi/15 **A1**

(a) Answer: 283pi/15

Question 16

(a) integrates: $4x^{-1}/(-1) = -4x^{-1}$ **M1**

(a) replaces the upper limit by t and lets t tend to infinity **M1**

(a) t^{-1} tends to 0 so the integral converges **A1**

(a) value = 4 **A1**

(a) Answer: 4

Question 17

(a) integrates: $-(5/5)e^{-5x}$ **M1**

(a) takes the limit as the upper bound tends to infinity **M1**

(a) 1 **A1**

(a) Answer: 1

Question 18

(a) V = π integral of e^{6x} dx **M1**

(a) integrates: $\pi[e^{6x}/6]$ **M1**

(a) substitutes the limits: $(\pi/6)(e^{18} - 1)$ **A1**

(a) Answer: $(\pi/6)(e^{18} - 1)$

Question 19

(a) V = π integral of y^2 dx **M1**

(a) π integral from 0 to 4 of x dx **A1**

(a) integrates: $\pi[1/2x^2]$ **M1**

(a) V = 8pi **A1**

(a) Answer: 8pi

Question 20

(a) mean value = (1/(b - a)) integral of f **M1**

(a) integral = 4 ln(3) **A1**

(a) mean value = ln(3) **A1**

(a) Answer: ln(3)

Question 21

(a) mean value = (1/(b - a)) integral of f **M1**

(a) integral = 3 ln(5) **A1**

(a) mean value = (3/4) ln(5) **A1**

(a) Answer: (3/4) ln(5)

Question 22

(a) uses the identity $r(r + 1)(r + 2) = [r(r + 1)(r + 2)(r + 3) - (r - 1)r(r + 1)(r + 2)]/4$ **M1**

(a) applies the method of differences so that intermediate terms cancel **M1**

(a) obtains $n(n + 1)(n + 2)(n + 3)/4$ (cso) **A1**

(a) Answer: $6n(n + 1)(n + 2)(n + 3)/4$

(b) S(15) - S(1) **M1**

(b) 110124 **A1**

(b) Answer: 110124

Question 23

uses $\sin^2(x) = (1-\cos 2x)/2$ to rewrite $3\sin^2(x)$ as $(3/2)(1-\cos 2x)$ **M1**

integrates to obtain $(3/2)(x - (1/2)\sin 2x)$ oe **M1**

substitutes the limits π and 0 correctly **M1**

integral from 0 to π of $3\sin^2(x)$ dx = $3*\pi/2$ oe **A1**

mean value = 3/2 cao **A1**

Answer: Mean value = 3/2

Question 24

(a) $\cos x = 1 - x^2/2$ oe **B1**

(a) $+ x^4/24$ oe **B1**

(a) valid for all real x **B1**

(a) Answer: $\cos x = 1 - x^2/2 + x^4/24 + ...$; valid for all real x

(b) substitutes x=0.5 into $1 - x^2/2 + x^4/24$ **M1**

(b) = 0.8776 (4 d.p.) cao **A1**

(b) Answer: 0.8776

(c) calculates |approximation - true value| / true value **M1**

(c) = 0.0028% awrt (2 s.f.) cao (allow 0.0023% if the 4 d.p. value 0.8776 from part (b) is used) **A1**

(c) Answer: 0.0028%

Question 25

(a) rearranges $y = \ln x$ to $x = e^y$ **M1**

(a) squares to obtain $x^2 = e^{2y}$ cso **A1**

(a) Answer: $x^2 = e^{2y}$ (shown)

(b) $V = \pi$ * integral from y=0 to y=1 of x^2 dy soi **B1**

(b) substitutes $x^2 = e^{2y}$ **M1**

(b) integrates e^{2y} to $(1/2)e^{2y}$ **M1**

(b) substitutes the limits y=1 and y=0 correctly **M1**

(b) $V = (e^2 - 1)*\pi/2$ oe cao **A1**

(b) Answer: $V = (e^2 - 1)\pi/2$

(c) V = 10.0 awrt (3 s.f.) **B1**

(c) Answer: 10.0 (awrt)

Question 26

(a) differentiates using the chain rule to obtain $f'(x) = 2/(1+2x)$ cso **M1**

(a) $f''(x) = -4/(1+2x)^2$ oe **A1**

(a) $f'''(x) = 16/(1+2x)^3$ oe **A1**

(a) Answer: $f'(x) = 2/(1+2x)$, $f''(x) = -4/(1+2x)^2$, $f'''(x) = 16/(1+2x)^3$

(b) evaluates f(0)=0, f'(0)=2, f''(0)=-4, f'''(0)=16 **M1**

(b) substitutes into $f(x) = f(0) + f'(0)x + f''(0)x^2/2! + f'''(0)x^3/3!$ **M1**

(b) $f(x) = 2x - 2x^2 + (8/3)x^3$ oe cao **A1**

(b) Answer: $\ln(1+2x) = 2x - 2x^2 + (8/3)x^3 + ...$

(c) $-1/2 < x \leq 1/2$ oe (allow $-1/2 < x < 1/2$ or $|x| < 1/2$) **B1**

(c) Answer: $-1/2 < x \leq 1/2$

Question 27

(a) writes f(r-1) = (r-1)r(r+1)(r+2) **M1**

(a) factorises f(r)-f(r-1) as r(r+1)(r+2)[(r+3)-(r-1)] **M1**

(a) = 4r(r+1)(r+2) cso **A1**

(a) Answer: f(r) - f(r-1) = 4r(r+1)(r+2) (shown)

(b) writes $\sum_{r=1}^{n} 4r(r+1)(r+2) = \sum_{r=1}^{n} [f(r)-f(r-1)]$ **M1**

(b) recognises the sum telescopes to f(n) - f(0) **M1**

(b) states f(0) = 0 **A1**

(b) divides both sides of 4*sum r(r+1)(r+2) = f(n) by 4 **M1**

(b) $\sum_{r=1}^{n} r(r+1)(r+2) = (1/4)n(n+1)(n+2)(n+3)$ cso **A1**

(b) Answer: $\sum_{r=1}^{n} r(r+1)(r+2) = (1/4)n(n+1)(n+2)(n+3)$

(c) substitutes n=15 into the formula **M1**

(c) = 18360 cao **A1**

(c) Answer: 18360

Question 28

(a) expands $r(2r-1)(r+3) = 2r^3 + 5r^2 - 3r$ **M1**

(a) writes the sum as $2*\text{sum } r^3 + 5*\text{sum } r^2 - 3*\text{sum } r$ **M1**

(a) substitutes the three standard results correctly **M1**

(a) factorises out n(n+1) from the resulting expression **M1**

(a) simplifies the bracket to $(3n^2+13n-4)/6$ **A1**

(a) sum = $n(n+1)(3n^2+13n-4)/6$ cso **A1**

(a) Answer: $\sum_{r=1}^{n} r(2r-1)(r+3) = n(n+1)(3n^2+13n-4)/6$

(b) substitutes n=12 into the formula **M1**

(b) = 15184 cao **A1**

(b) Answer: 15184

Question 29

(a) uses $V(b) = \pi$ * integral from x=1 to x=b of y^2 dx with $y^2 = 1/x^2$ **M1**

(a) integrates x^{-2} to $-1/x$ **M1**

(a) substitutes the limits b and 1 correctly **M1**

(a) $V(b) = \pi(1 - 1/b)$ cso **A1**

(a) Answer: $V(b) = \pi(1 - 1/b)$ (shown)

(b) considers the limit as b -> infinity of 1/b **M1**

(b) 1/b -> 0 as b -> infinity **A1**

(b) V(b) -> π, so the volume converges to π cao **A1**

(b) Answer: The volume converges to π

(c) recognises that S has a finite volume (π, from part (b)) but an infinite surface area (since A(b) -> infinity) **B1**

(c) gives a correct comment, e.g. that a solid can enclose a finite volume even though its bounding surface has infinite area, so finite volume does not imply finite surface area **B1**

(c) Answer: S has finite volume π but infinite surface area; a solid can have a finite volume while its surface area is unbounded.

(d) uses $V = \pi$ * integral from x=1 to x=10 of $(1/x)^2$ dx **M1**

(d) integrates and substitutes the limits 10 and 1 correctly **M1**

(d) $V = 9*\pi/10$ cao **A1**

(d) Answer: V = 9pi/10

Question 30

(a) sums both sides of the given identity from r=1 to n, giving 2sin(1/2)*sum cos(r) = sum [sin(r+1/2) - sin(r-1/2)] **M1**

(a) lists the first two and last two terms of the right-hand sum to show the telescoping pattern **M1**

(a) the right-hand sum telescopes to sin(n+1/2) - sin(1/2) **A1**

(a) equates 2sin(1/2)*sum cos(r) to this telescoped result **M1**

(a) divides both sides by 2sin(1/2), valid since sin(1/2) is not 0 **B1**

(a) $\sum_{r=1}^{n} \cos(r) = (\sin(n+1/2) - \sin(1/2))/(2\sin(1/2))$ cso **A1**

(a) Answer: $\sum_{r=1}^{n} \cos(r) = (\sin(n+1/2) - \sin(1/2))/(2\sin(1/2))$

(b) substitutes n=50 into the formula from part (a) **M1**

(b) evaluates sin(50.5) and sin(0.5) correctly (in radians) **M1**

(b) = -0.258 awrt (3 s.f.) cao **A1**

(b) Answer: -0.258