1.15 Core Pure: Polar Coordinates and Hyperbolic Functions Depth: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has polar equation $r = 4 + 4\cos(\theta)$, for $0 \leq \theta < 2\pi$.

(a) Find the area enclosed by C. **(6)**

(b) Find the area of the region that lies inside C and outside the circle with polar equation $r = 4$. **(7)**

(Total for Question 1 is 13 marks)

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2 This question is about integrating hyperbolic and related functions.

(a) Find the exact value of the integral from 0 to 25/12 of $1/\sqrt{x^2 + 25}$ dx, giving your answer as a natural logarithm. **(4)**

(Total for Question 2 is 4 marks)

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3 It is given that $\sinh x = 5/12$, where x is real.

(a) Use a hyperbolic identity to find the exact value of $\cosh x$, and hence $\tanh x$. **(4)**

(b) Find the exact value of x, giving your answer as a natural logarithm. **(3)**

(c) Find the exact value of $\sinh 2x$ and of $\cosh 2x$. **(3)**

(Total for Question 3 is 10 marks)

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4 The curve C has equation $y = 8\cosh x - 3\sinh x$.

(a) Find dy/dx and show that C has a stationary point at $x = (1/2)\ln(11/5)$. **(4)**

(b) Find the exact y coordinate of the stationary point, and determine its nature. **(3)**

(Total for Question 4 is 7 marks)

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5 The curve C has polar equation $r = 6(1 + \cos\theta)$, for $-\pi < \theta \leq \pi$. The region R is bounded by C, the initial line and the half line $\theta = \pi/2$.

(a) Find the exact area of R. **(4)**

(Total for Question 5 is 4 marks)

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6 Solve the equation $43 \cosh x - 41 \sinh x = 13$, giving your answers as exact logarithms.

(a) Show that the equation can be written as $2e^{2x} - 26e^{x} + 84 = 0$. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 6 is 7 marks)

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7 It is given that $\sinh x = 15/8$, where x is real.

(a) Use a hyperbolic identity to find the exact value of $\cosh x$, and hence $\tanh x$. **(4)**

(b) Find the exact value of x, giving your answer as a natural logarithm. **(3)**

(c) Find the exact value of $\sinh 2x$ and of $\cosh 2x$. **(3)**

(Total for Question 7 is 10 marks)

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Mark scheme · 1.15 Core Pure: Polar Coordinates and Hyperbolic Functions Depth: 1-hour Pack

Question 1

(a) Area = (1/2) times the integral of r^2 with respect to θ, with limits 0 to 2pi **M1**

(a) expand $(4+4\cos(\theta))^2 = 16 + 32\cos(\theta) + 16\cos^2(\theta)$ **M1**

(a) use $\cos^2(\theta) = (1+\cos(2\theta))/2$ to rewrite the integrand **M1**

(a) correct integral: (1/2) times the integral of (16 + 32cos(θ) + 8 + 8cos(2theta)) dtheta **A1**

(a) integrate term by term and substitute the limits 0 to 2pi (the cos(θ) and cos(2theta) terms integrate to zero over a full period) **M1**

(a) cao: Area = 24pi **A1**

(a) Answer: Area = 24pi

(b) find the points of intersection: 4+4cos(θ) = 4 gives cos(θ) = 0, so θ = +-π/2 **M1**

(b) set up Area = (1/2) times the integral, with limits -π/2 to π/2, of $[(4+4\cos(\theta))^2 - 4^2]$ dtheta **M1**

(b) expand the integrand to $32\cos(\theta) + 16\cos^2(\theta)$ **M1**

(b) use $\cos^2(\theta) = (1+\cos(2\theta))/2$ to write the integrand as $32\cos(\theta) + 8 + 8\cos(2\theta)$ **A1**

(b) integrate to obtain 32sin(θ) + 8theta + 4sin(2theta) **dM1**

(b) correct evaluation between the limits gives 64 + 8pi **A1**

(b) cao: Area = (1/2)(64+8pi) = 32 + 4pi **A1**

(b) Answer: Area = 32 + 4pi

Question 2

(a) uses the standard integral: arsinh(x/5) **M1**

(a) arsinh(5/12) = $\ln(5/12 + \sqrt{1 + 25/144})$ **A1**

(a) $\sqrt{1 + 25/144} = 13/12$ **A1**

(a) ln(3/2) **A1**

(a) Answer: ln(3/2)

Question 3

(a) uses $\cosh^2 x - \sinh^2 x = 1$ **M1**

(a) $\cosh^2 x = 1 + 25/144 = 169/144$ **A1**

(a) cosh x = 13/12 (positive root, since cosh x ≥ 1) **A1**

(a) tanh x = 5/13 **A1**

(a) Answer: cosh x = 13/12, tanh x = 5/13

(b) uses the definition $\sinh x = (e^x - e^{-x})/2$ or the logarithmic form of arsinh **M1**

(b) $e^x = \sinh x + \cosh x = 3/2$ **A1**

(b) x = ln(3/2) **A1**

(b) Answer: x = ln(3/2)

(c) uses $\sinh 2x = 2\sinh x \cosh x$ and $\cosh 2x = 1 + 2\sinh^2 x$ **M1**

(c) sinh 2x = 65/72 **A1**

(c) cosh 2x = 97/72 **A1**

(c) Answer: sinh 2x = 65/72, cosh 2x = 97/72

Question 4

(a) dy/dx = 8 sinh x - 3 cosh x **M1**

(a) sets dy/dx = 0: tanh x = 3/8 **M1**

(a) x = artanh(3/8) = (1/2) ln((1 + 3/8)/(1 - 3/8)) **A1**

(a) = (1/2) ln(11/5) (cso) **A1**

(a) Answer: x = (1/2) ln(11/5)

(b) substitutes: cosh x = $8/\sqrt{55}$ etc, or writes y = $\sqrt{55}$ cosh(x - c) **M1**

(b) y = $\sqrt{55}$ **A1**

(b) $d^2y/dx^2 = y > 0$ at the stationary point, so a minimum **B1**

(b) Answer: y = $\sqrt{55}$ (minimum)

Question 5

(a) area = (1/2) integral from 0 to π/2 of r^2 d θ **M1**

(a) uses $\cos^2 \theta = (1 + \cos 2\theta)/2$ and integrates **M1**

(a) substitutes the limits 0 and π/2 **M1**

(a) 36(3pi/8 + 1) **A1**

(a) Answer: 36(3pi/8 + 1)

Question 6

(a) writes $\cosh x = (e^x + e^{-x})/2$ and $\sinh x = (e^x - e^{-x})/2$ **M1**

(a) multiplies through by e^x and collects terms **M1**

(a) $2e^{2x} - 26e^x + 84 = 0$ (cso) **A1**

(a) Answer: $2e^{2x} - 26e^x + 84 = 0$

(b) treats as a quadratic in e^x and factorises **M1**

(b) $e^x = 6$ or $e^x = 7$ **A1**

(b) takes natural logs **M1**

(b) x = ln 6 and x = ln 7 **A1**

(b) Answer: x = ln 6, x = ln 7

Question 7

(a) uses $\cosh^2 x - \sinh^2 x = 1$ **M1**

(a) $\cosh^2 x = 1 + 225/64 = 289/64$ **A1**

(a) cosh x = 17/8 (positive root, since cosh x ≥ 1) **A1**

(a) tanh x = 15/17 **A1**

(a) Answer: cosh x = 17/8, tanh x = 15/17

(b) uses the definition $\sinh x = (e^x - e^{-x})/2$ or the logarithmic form of arsinh **M1**

(b) $e^x = \sinh x + \cosh x = 4$ **A1**

(b) x = ln 4 **A1**

(b) Answer: x = ln 4

(c) uses $\sinh 2x = 2\sinh x \cosh x$ and $\cosh 2x = 1 + 2\sinh^2 x$ **M1**

(c) sinh 2x = 255/32 **A1**

(c) cosh 2x = 257/32 **A1**

(c) Answer: sinh 2x = 255/32, cosh 2x = 257/32

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