1.15 Core Pure: Polar Coordinates and Hyperbolic Functions Depth: 30-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question concerns the inverse hyperbolic function artanh.

(a) Given that y = artanh(x) for |x| < 1, so that x = tanh(y), show that artanh(x) = (1/2)ln((1+x)/(1-x)). **(5)**

(b) Hence solve, for x in the real numbers, the equation 3tanh(x) = 2, giving your answer as an exact single logarithm. **(3)**

(c) Differentiate artanh(2x) with respect to x, simplifying your answer. **(3)**

(Total for Question 1 is 11 marks)

---

2 This question tests differentiation and integration of hyperbolic functions.

(a) Differentiate $\cosh(1 - 4x^2)$ with respect to x. **(2)**

(b) Differentiate ln(sinh(3x)) with respect to x, for x > 0, simplifying your answer. **(3)**

(c) Differentiate $x^2 \tanh(x)$ with respect to x. **(3)**

(d) Find, in simplest form, the integral of $x \cosh(x^2)$ with respect to x. **(2)**

(e) Find the integral of tanh(x) with respect to x, giving your answer in the form ln(f(x)) + c. **(3)**

(Total for Question 2 is 13 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.15 Core Pure: Polar Coordinates and Hyperbolic Functions Depth: 30-minute Pack

Question 1

(a) write $x = \tanh(y) = (e^{2y}-1)/(e^{2y}+1)$, obtained by multiplying numerator and denominator of $(e^{y}-e^{-y})/(e^{y}+e^{-y})$ by e^{y}	**M1**
(a) form the equation $x(e^{2y}+1) = e^{2y} - 1$	**M1**
(a) rearrange to isolate e^{2y}: $x + 1 = e^{2y}(1-x)$	**M1**
(a) $e^{2y} = (1+x)/(1-x)$	**A1**
(a) cso: $y = (1/2)\ln((1+x)/(1-x))$	**A1**
(a) Answer: artanh(x) = (1/2)ln((1+x)/(1-x))	
(b) $\tanh(x) = 2/3$, substituted into $x = (1/2)\ln((1+x)/(1-x))$ with $x=2/3$	**M1**
(b) simplify $(1+2/3)/(1-2/3) = (5/3)/(1/3) = 5$	**A1**
(b) cao: $x = (1/2)\ln(5)$ $(= \ln(\sqrt{5}))$	**A1**
(b) Answer: $x = (1/2)\ln(5) = \ln(\sqrt{5})$	
(c) use $d/dx[\text{artanh}(u)] = u'/(1-u^2)$ with $u = 2x$	**M1**
(c) correct substitution $u=2x$, $u'=2$	**A1**
(c) cao: $d/dx[\text{artanh}(2x)] = 2/(1-4x^2)$	**A1**
(c) Answer: $d/dx[\text{artanh}(2x)] = 2/(1-4x^2)$	

Question 2

(a) apply the chain rule: derivative of cosh(u) is sinh(u) times u', with $u = 1-4x^2$, $u' = -8x$	**M1**
(a) cao: $-8x \sinh(1-4x^2)$	**A1**
(a) Answer: $-8x \sinh(1 - 4x^2)$	
(b) apply the chain rule: (1/sinh(3x)) times derivative of sinh(3x)	**M1**
(b) derivative of $\sinh(3x) = 3\cosh(3x)$ used correctly	**A1**
(b) cao: simplify $(3\cosh(3x))/\sinh(3x) = 3\coth(3x)$	**A1**
(b) Answer: 3coth(3x)	
(c) apply the product rule u'v + uv' with $u = x^2$, $v = \tanh(x)$	**M1**
(c) use derivative of $\tanh(x) = \text{sech}^2(x)$ correctly	**A1**
(c) cao: $2x \tanh(x) + x^2 \text{sech}^2(x)$	**A1**
(c) Answer: $2x \tanh(x) + x^2 \text{sech}^2(x)$	
(d) recognise $x \cosh(x^2)$ as (1/2) times the derivative of $\sinh(x^2)$	**M1**
(d) cao: $(1/2)\sinh(x^2) + c$	**A1**
(d) Answer: $(1/2)\sinh(x^2) + c$	
(e) write $\tanh(x) = \sinh(x)/\cosh(x)$ and recognise the numerator as the derivative of the denominator	**M1**
(e) integral form $\ln\lvert\cosh(x)\rvert + c$	**A1**
(e) cao: since $\cosh(x) > 0$ for all real x, this simplifies to $\ln(\cosh(x)) + c$	**A1**
(e) Answer: $\ln(\cosh(x)) + c$	

© Revision Library | revisionlibrary.co.uk