1.15 Core Pure: Polar Coordinates and Hyperbolic Functions Depth: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has polar equation $r = 6(\theta)$, for $\theta \geq 0$ (an Archimedean spiral). The point P lies on C where $\theta = \pi/3$.

(a) Find the exact Cartesian coordinates of P. **(4)**

(b) Hence write down the exact distance OP, where O is the pole. **(1)**

(Total for Question 1 is 5 marks)

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2 You are given that $\sinh(A+B) = \sinh(A)\cosh(B) + \cosh(A)\sinh(B)$ and $\cosh(A+B) = \cosh(A)\cosh(B) + \sinh(A)\sinh(B)$ for all real A and B.

(a) Prove that $\tanh(A+B) = (\tanh(A) + \tanh(B)) / (1 + \tanh(A)\tanh(B))$. **(5)**

(b) Given that $\tanh(x) = 1/3$, use the result from part (a) to find the exact value of $\tanh(2x)$. **(4)**

(Total for Question 2 is 9 marks)

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3 The curve C has polar equation $r^2 = 12\cos(2\text{theta})$, for $-\pi/4 \leq \theta \leq \pi/4$ (one loop of a lemniscate).

(a) Write down the values of θ, in the given interval, for which $r = 0$. **(1)**

(b) Show that the area of the loop of C described is 6. **(5)**

(c) Write down the total area enclosed by both loops of the full curve $r^2 = 12\cos(2\text{theta})$, justifying your answer briefly. **(2)**

(Total for Question 3 is 8 marks)

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4 The curve C has polar equation $r = 2 + 2\sin(\theta)$, for $0 \leq \theta < 2\text{pi}$.

(a) Show that $dy/d\text{theta} = 2\cos(\theta)(1 + 2\sin(\theta))$, where $y = r*\sin(\theta)$. **(3)**

(b) Find, in exact form, the values of θ in the interval $0 \leq \theta < 2\text{pi}$ for which the tangent to C is parallel to the initial line, explaining why $\theta = 3\text{pi}/2$ must be rejected. **(6)**

(Total for Question 4 is 9 marks)

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5 The curve C has polar equation $r = 6(2 + \cos\theta)$, for $0 \leq \theta < 2\pi$.

(a) Find the exact area enclosed by C. **(4)**

(Total for Question 5 is 4 marks)

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6 The curve C has equation $y = 7\cosh x - 2\sinh x$.

(a) Find dy/dx and show that C has a stationary point at $x = (1/2)\ln(9/5)$. **(4)**

(b) Find the exact y coordinate of the stationary point, and determine its nature. **(3)**

(Total for Question 6 is 7 marks)

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7 The curve C has polar equation $r = 1 + 3\cos(\theta)$, for $0 \leq \theta < 2\pi$. Since the coefficient of $\cos(\theta)$ (3) is greater than the constant term (1), C has an inner loop.

(a) Find, to 3 significant figures, the values of θ in the interval $0 \leq \theta < 2\pi$ for which $r = 0$. **(2)**

(b) Show that $dx/d\text{theta} = -\sin(\theta)(1 + 6\cos(\theta))$, where $x = r*\cos(\theta)$, and hence find, to 3 significant figures, the values of θ in the interval $0 \leq \theta < \pi$ for which the tangent to C is perpendicular to the initial line. **(8)**

(Total for Question 7 is 10 marks)

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Mark scheme · 1.15 Core Pure: Polar Coordinates and Hyperbolic Functions Depth: Test Pack

Question 1

(a) find r = 6(π/3) = 2pi at θ = π/3 **M1**

(a) use the exact values cos(π/3) = 1/2 and sin(π/3) = $\sqrt{3}$/2 **M1**

(a) x = r*cos(π/3) = 2pi*(1/2) = π **A1**

(a) y = r*sin(π/3) = 2pi*($\sqrt{3}$/2) = π*$\sqrt{3}$ **A1**

(a) Answer: P = (π, π*$\sqrt{3}$)

(b) OP = r = 2pi (the distance from the pole is simply the value of r at that θ) **B1**

(b) Answer: OP = 2pi

Question 2

(a) write tanh(A+B) = sinh(A+B)/cosh(A+B) and substitute the given addition formulae **M1**

(a) divide every term in the numerator and denominator by cosh(A)cosh(B) **M1**

(a) numerator simplifies to tanh(A) + tanh(B), using sinh(A)/cosh(A) = tanh(A) **A1**

(a) denominator simplifies to 1 + tanh(A)tanh(B), using sinh(A)sinh(B)/(cosh(A)cosh(B)) = tanh(A)tanh(B) **A1**

(a) cso: tanh(A+B) = (tanh(A)+tanh(B))/(1+tanh(A)tanh(B)) **A1**

(a) Answer: tanh(A+B) = (tanh(A)+tanh(B)) / (1+tanh(A)tanh(B)), proved.

(b) use tanh(2x) = 2tanh(x)/(1+tanh^2(x)), the case A=B=x of part (a) **M1**

(b) substitute tanh(x) = 1/3 **M1**

(b) simplify (2/3)/(1+1/9) = (2/3)/(10/9) **A1**

(b) cao: tanh(2x) = 3/5 **A1**

(b) Answer: tanh(2x) = 3/5

Question 3

(a) θ = -π/4 and θ = π/4 (both required) **B1**

(a) Answer: θ = -π/4, π/4

(b) Area = (1/2) times the integral of r^2 with respect to θ, with limits -π/4 to π/4 **M1**

(b) correct integral: (1/2) times the integral of 12cos(2theta) dtheta **A1**

(b) integrate to obtain 3sin(2theta) (or equivalent antiderivative) **M1**

(b) substitute the limits: 3[sin(π/2) - sin(-π/2)] **M1**

(b) cao: Area = 3[1-(-1)] = 6 **A1**

(b) Answer: Area = 6 (square units)

(c) recognise that the second loop (for θ between 3pi/4 and 5pi/4) is congruent to the first by symmetry **B1**

(c) cao: total area = 2 x 6 = 12 **B1**

(c) Answer: Total area = 12 (square units)

Question 4

(a) write y = (2+2sin(θ))sin(θ) = 2sin(θ) + 2sin^2(θ) **M1**

(a) differentiate to get dy/dtheta = 2cos(θ) + 4sin(θ)cos(θ) **M1**

(a) cso: factorise to dy/dtheta = 2cos(θ)(1 + 2sin(θ)) **A1**

(a) Answer: dy/dtheta = 2cos(θ)(1 + 2sin(θ))

(b) set dy/dtheta = 0 **M1**

(b) solve cos(θ) = 0 to get θ = π/2 or θ = 3pi/2 **M1**

(b) solve sin(θ) = -1/2 to get θ = 7pi/6 or θ = 11pi/6 **M1**

(b) all four values π/2, 3pi/2, 7pi/6, 11pi/6 correctly identified as solutions of dy/dtheta = 0 **A1**

(b) recognise that r = 2+2sin(θ) = 0 at θ = 3pi/2, so C passes through the pole there and the tangent direction is not given by this stationary-point condition, so θ = 3pi/2 is rejected **B1**

(b) cao: θ = π/2, 7pi/6, 11pi/6 **A1**

(b) Answer: θ = π/2, 7pi/6, 11pi/6

Question 5

(a) area = (1/2) integral from 0 to 2pi of r^2 d θ **M1**

(a) expands 36(4 + 4cos θ + cos^2 θ) **M1**

(a) integrates, using cos^2 θ = (1 + cos 2theta)/2, and substitutes the limits **M1**

(a) 324pi/2 **A1**

(a) Answer: 324pi/2

Question 6

(a) dy/dx = 7 sinh x - 2 cosh x **M1**

(a) sets dy/dx = 0: tanh x = 2/7 **M1**

(a) x = artanh(2/7) = (1/2) ln((1 + 2/7)/(1 - 2/7)) **A1**

(a) = (1/2) ln(9/5) (cso) **A1**

(a) Answer: x = (1/2) ln(9/5)

(b) substitutes: cosh x = 7/$\sqrt{45}$ etc, or writes y = $\sqrt{45}$ cosh(x - c) **M1**

(b) y = 3$\sqrt{5}$ **A1**

(b) d^{2}y/dx^2 = y > 0 at the stationary point, so a minimum **B1**

(b) Answer: y = 3$\sqrt{5}$ (minimum)

Question 7

(a) solve cos(θ) = -1/3 **B1**

(a) θ = 1.91 or θ = 4.37 (both awrt 3 s.f.) **B1**

(a) Answer: θ = 1.91 or θ = 4.37 (radians, 3 s.f.)

(b) write x = (1+3cos(θ))cos(θ) = cos(θ) + 3cos^2(θ) **M1**

(b) differentiate to get dx/dtheta = -sin(θ) - 6sin(θ)cos(θ) **M1**

(b) cso: factorise to dx/dtheta = -sin(θ)(1 + 6cos(θ)) **A1**

(b) set dx/dtheta = 0 and split into sin(θ) = 0 or cos(θ) = -1/6 **M1**

(b) θ = 0 is a solution in the given range (from sin(θ)=0; θ=π is excluded as the interval is 0≤θ<π) **A1**

(b) solve cos(θ) = -1/6 using a calculator **M1**

(b) θ = 1.74 (3 s.f.) **A1**

(b) confirm r is non-zero at both θ=0 (r=4) and θ=1.74 (r=0.5), so both give genuine tangent lines (neither coincides with the pole angles from part (a)) **B1**

(b) Answer: θ = 0 or θ = 1.74 (3 s.f.)

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