1.2 Matrices: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The matrices A and B are given by A = ((1, -4), (-3, 3)) and B = ((-2, 4), (1, -1)), where each matrix is written row by row.

(a) Find AB. **(2)**

(b) Find det A and, given that A is non-singular, find A^{-1}. **(4)**

(c) Use A^{-1} to solve the equations x - 4y = 11 and -3x + 3y = -15. **(3)**

(Total for Question 1 is 9 marks)

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2 The matrices A and B are given by A = ((-1, 3), (0, -1)) and B = ((2, -3), (3, 4)), where each matrix is written row by row.

(a) Find AB. **(2)**

(b) Find det A and, given that A is non-singular, find A^{-1}. **(4)**

(c) Use A^{-1} to solve the equations -x + 3y = 15 and -y = -4. **(3)**

(Total for Question 2 is 9 marks)

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3 The matrix M is given by M = ((-5, 2), (0, 2)).

(a) Find the characteristic equation of M and hence find its eigenvalues. **(3)**

(b) Find an eigenvector corresponding to each eigenvalue, giving each as a vector with integer components in simplest form. **(3)**

(c) Find M^2. **(2)**

(Total for Question 3 is 8 marks)

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4 The transformation T_1 is a reflection in the line y = -x. The transformation T_2 is an enlargement with scale factor 2, centre the origin.

(a) Write down the 2 x 2 matrices M_1 and M_2 representing T_1 and T_2. **(2)**

(b) Find the matrix representing the transformation T_1 followed by T_2. **(2)**

(c) Find the image of the point (-4, 2) under T_1 followed by T_2. **(2)**

(d) A triangle of area 6 is transformed by T_1 followed by T_2. Find the area of its image. **(2)**

(Total for Question 4 is 8 marks)

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5 A transformation of the plane is represented by the matrix M = [[5,-2],[4,-1]].

(a) Show that every point on the line y = 2x is an invariant point of the transformation represented by M. **(3)**

(b) Find the equation of the other invariant line of M through the origin (not consisting entirely of invariant points). **(4)**

(c) State the scale factor of enlargement associated with this second invariant line. **(1)**

(d) Verify your answer to part (b) by finding the image of the point (1,1) under M and confirming it lies on the same invariant line, stating the scale factor this confirms. **(2)**

(Total for Question 5 is 10 marks)

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6 A system of equations is given by x + 2y - z = 3, 2x + y + z = 5, 3x + 3y + (λ)z = μ, where λ and μ are real constants. The coefficient matrix is A(λ) = [[1,2,-1],[2,1,1],[3,3,λ]].

(a) Show that det(A(λ)) = -3*λ. **(3)**

(b) State the value of λ for which the system does not have a unique solution. **(1)**

(c) When λ = 0, find the value of μ for which the system is consistent. **(3)**

(d) Describe geometrically the configuration of the three planes when λ = 0, in the two cases (i) μ = 8 and (ii) μ is not equal to 8. **(3)**

(Total for Question 6 is 10 marks)

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Mark scheme · 1.2 Matrices: 1-hour Pack

Question 1

(a) multiplies rows of A by columns of B **M1**

(a) ((-6, 8), (9, -15)) **A1**

(a) Answer: ((-6, 8), (9, -15))

(b) det A = ad - bc **M1**

(b) det A = -9 **A1**

(b) uses the inverse formula (1/det)((d, -b), (-c, a)) **M1**

(b) A^{-1} = ((-1/3, -4/9), (-1/3, -1/9)) **A1**

(b) Answer: det A = -9, A^{-1} = ((-1/3, -4/9), (-1/3, -1/9))

(c) writes as a matrix equation A(x, y) = (b1, b2) and multiplies by A^{-1} **M1**

(c) x = 3 **A1**

(c) y = -2 **A1**

(c) Answer: x = 3, y = -2

Question 2

(a) multiplies rows of A by columns of B **M1**

(a) ((7, 15), (-3, -4)) **A1**

(a) Answer: ((7, 15), (-3, -4))

(b) det A = ad - bc **M1**

(b) det A = 1 **A1**

(b) uses the inverse formula (1/det)((d, -b), (-c, a)) **M1**

(b) A^{-1} = ((-1, -3), (0, -1)) **A1**

(b) Answer: det A = 1, A^{-1} = ((-1, -3), (0, -1))

(c) writes as a matrix equation A(x, y) = (b1, b2) and multiplies by A^{-1} **M1**

(c) x = -3 **A1**

(c) y = 4 **A1**

(c) Answer: x = -3, y = 4

Question 3

(a) forms det(M - λ I) = 0 **M1**

(a) λ^2 + 3lambda - 10 = 0 **A1**

(a) λ = -5 and λ = 2 **A1**

(a) Answer: λ = -5, 2

(b) solves (M - λ I) v = 0 for the first eigenvalue **M1**

(b) for λ = -5: (1, 0) **A1**

(b) for λ = 2: (2, 7) **A1**

(b) Answer: λ = -5: (1, 0); λ = 2: (2, 7)

(c) multiplies M by itself 2 times (or uses the diagonalisation PD^2P^{-1}) **M1**

(c) ((25, -6), (0, 4)) **A1**

(c) Answer: ((25, -6), (0, 4))

Question 4

(a) M_1 = ((0, -1), (-1, 0)) **B1**

(a) M_2 = ((2, 0), (0, 2)) **B1**

(a) Answer: M_1 = ((0, -1), (-1, 0)), M_2 = ((2, 0), (0, 2))

(b) multiplies in the correct order: M_2 M_1 **M1**

(b) ((0, -2), (-2, 0)) **A1**

(b) Answer: ((0, -2), (-2, 0))

(c) multiplies the combined matrix by the position vector **M1**

(c) (-4, 8) **A1**

(c) Answer: (-4, 8)

(d) area scale factor = |det of the combined matrix| **M1**

(d) |-4| x 6 = 24 **A1**

(d) Answer: 24

Question 5

(a) taking a general point (x, 2x) on the line and forming $M(x,2x)^T$ **M1**

(a) correctly computing both components: 5x - 2(2x) = x and 4x - 1(2x) = 2x **M1**

(a) cso: image (x,2x) equals original point, so every point on y=2x is invariant **A1**

(a) Answer: Shown: $M(x,2x)^T = (x,2x)^T$ for all x, so y = 2x is a line of invariant points.

(b) considering a general point (x,mx) on line y=mx and forming $M(x,mx)^T$ = (x(5-2m), x(4-m)) **M1**

(b) setting 4-m = m(5-2m) to require the image to lie on y=mx **M1**

(b) forming and solving the quadratic $2m^2$ - 6m + 4 = 0 oe, i.e. m^2-3m+2=0 **M1**

(b) m = 2 (already found in (a)) or m = 1; other invariant line is y = x cao **A1**

(b) Answer: y = x

(c) scale factor 3 (ft their line if consistent), oe **B1**

(c) Answer: Scale factor 3

(d) $M(1,1)^T$ = (3,3) found correctly (ft their M) **M1**

(d) (3,3) lies on y=x, and (3,3) = 3*(1,1), confirming scale factor 3 cao **A1**

(d) Answer: M(1,1) = (3,3), which lies on y=x, confirming scale factor 3.

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Question 6

(a) correct expansion of det(A(λ)) along a row or column **M1**

(a) correct simplification of all three terms **A1**

(a) cso: det(A(λ)) = -3*λ (answer printed) **A1**

(a) Answer: det(A(λ)) = -3*λ

(b) λ = 0, ft their part (a) **B1**

(b) Answer: λ = 0

(c) adding equation 1 and equation 2 to eliminate z: 3x + 3y = 8 **M1**

(c) comparing with equation 3 at λ = 0: 3x + 3y = μ **M1**

(c) μ = 8 cao **A1**

(c) Answer: μ = 8

(d) (i) the three planes meet in a common line (a sheaf of planes), giving infinitely many solutions **B1**

(d) (ii) the planes have no common point **B1**

(d) (ii) they form a triangular prism, each pair of planes meeting in a line, with the three lines parallel and distinct oe **B1**

(d) Answer: (i) μ = 8: planes meet in a common line, infinitely many solutions. (ii) μ ≠ 8: planes form a triangular prism, no common point, system inconsistent.

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