# 1.2 Matrices: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 70 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions. Show all your working.**

**1** Matrices A and B are given by A = [[2,-1],[3,4]] and B = [[0,5],[-2,1]].

**(a)** Find A + B. **(1)**

**(b)** Find 3A - 2B. **(2)**

**(c)** Find the matrix product AB. **(3)**

**(d)** Find the matrix product BA and hence state, with a reason, whether matrix multiplication is commutative for A and B. **(3)**

**(Total for Question 1 is 9 marks)**

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**2** The transformation $T_1$ is a stretch with scale factor 3 parallel to the y axis. The transformation $T_2$ is an enlargement with scale factor 2, centre the origin.

**(a)** Write down the 2 x 2 matrices $M_1$ and $M_2$ representing $T_1$ and $T_2$. **(2)**

**(b)** Find the matrix representing the transformation $T_1$ followed by $T_2$. **(2)**

**(c)** Find the image of the point (2, -1) under $T_1$ followed by $T_2$. **(2)**

**(d)** A triangle of area 6 is transformed by $T_1$ followed by $T_2$. Find the area of its image. **(2)**

**(Total for Question 2 is 8 marks)**

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**3** The matrix A is given by A = ((k, 2, 3), (-1, -2, 1), (0, 3, -3)), written row by row, where k is a constant.

**(a)** Show that det A = 3k - 15. **(3)**

**(b)** Find the value of k for which A is singular. **(2)**

**(c)** State the value of det A when k = -3, and hence state whether A is invertible when k = -3. **(2)**

**(Total for Question 3 is 7 marks)**


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**4** P = [[k,1,1],[1,k,1],[1,1,k]], where k is a real constant.

**(a)** Show that det(P) = $k^3 - 3k + 2$. **(4)**

**(b)** Show that det(P) = $(k-1)^2(k+2)$. **(2)**

**(c)** Find the value(s) of k for which P is singular. **(2)**

**(Total for Question 4 is 8 marks)**

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**5** A fairground in Weymouth charges different prices, in pounds, for adult (a), child (c) and senior (s) tickets. Ticket sales over three days are recorded as: Day 1: 2 adult + 3 child + 1 senior = £34. Day 2: 1 adult + 2 child + 2 senior = £24. Day 3: 3 adult + 1 child + 1 senior = £32.

**(a)** Write this information as a matrix equation Qv = r, where $v = (a,c,s)^T$, stating Q and r. **(1)**

**(b)** Find det(Q). **(3)**

**(c)** Find $Q^{-1}$. **(4)**

**(d)** Hence find the price of an adult, a child and a senior ticket. **(3)**

**(Total for Question 5 is 11 marks)**

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**6** A transformation of the plane is represented by the matrix W = [[7,-6],[3,-2]].

**(a)** Find the eigenvalues of W by solving det(W - λ*I) = 0. **(4)**

**(b)** By considering a general line y = mx through the origin, find the equations of the two invariant lines of the transformation represented by W. **(5)**

**(c)** Determine which of the two invariant lines consists entirely of invariant points, justifying your answer using the eigenvalues from part (a). **(2)**

**(Total for Question 6 is 11 marks)**


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# Mark scheme · 1.2 Matrices: Test Pack

## Question 1

(a) A + B = [[2,4],[1,5]] cao **B1**

**(a) Answer: A + B = [[2,4],[1,5]]**

(b) 3A = [[6,-3],[9,12]] or 2B = [[0,10],[-4,2]] found correctly **M1**

(b) 3A - 2B = [[6,-13],[13,10]] cao **A1**

**(b) Answer: 3A - 2B = [[6,-13],[13,10]]**

(c) attempt at matrix multiplication with at least one correct entry **M1**

(c) two correct entries of AB **A1**

(c) AB = [[2,9],[-8,19]] cao **A1**

**(c) Answer: AB = [[2,9],[-8,19]]**

(d) attempt at matrix multiplication BA with at least one correct entry **M1**

(d) BA = [[15,20],[-1,6]] cao **A1**

(d) correct conclusion that AB is not equal to BA (ft their AB, BA), so matrix multiplication is not commutative here oe **B1**

**(d) Answer: BA = [[15,20],[-1,6]]; since AB = [[2,9],[-8,19]] is not equal to BA, multiplication is not commutative.**

## Question 2

(a) $M_1$ = ((1, 0), (0, 3)) **B1**

(a) $M_2$ = ((2, 0), (0, 2)) **B1**

**(a) Answer: $M_1$ = ((1, 0), (0, 3)), $M_2$ = ((2, 0), (0, 2))**

(b) multiplies in the correct order: $M_2$ $M_1$ **M1**

(b) ((2, 0), (0, 6)) **A1**

**(b) Answer: ((2, 0), (0, 6))**

(c) multiplies the combined matrix by the position vector **M1**

(c) (4, -6) **A1**

**(c) Answer: (4, -6)**

(d) area scale factor = |det of the combined matrix| **M1**

(d) |12| x 6 = 72 **A1**

**(d) Answer: 72**

## Question 3

(a) expands the determinant along a row or column **M1**

(a) evaluates the 2 x 2 minors correctly **A1**

(a) det A = 3k - 15 (cso) **A1**

**(a) Answer: det A = 3k - 15**

(b) sets det A = 0 **M1**

(b) k = 5 **A1**

**(b) Answer: k = 5**

(c) det A = -24 **B1**

(c) invertible **B1**

**(c) Answer: -24; invertible**

## Question 4

(a) correct expansion along a row or column: $k(k^2-1) - 1(k-1) + 1(1-k)$ oe **M1**

(a) each of the three 2x2 minors correct **A1**

(a) correct simplification of the expansion **M1**

(a) cso: det(P) = $k^3 - 3k + 2$ (answer printed) **A1**

**(a) Answer: det(P) = $k^3 - 3k + 2$**

(b) identifying k=1 as a root and factorising out (k-1), or equivalent factorisation attempt **M1**

(b) cso: $(k-1)^2(k+2)$ confirmed by expansion matching $k^3-3k+2$ **A1**

**(b) Answer: det(P) = $(k-1)^2(k+2)$**

(c) setting $(k-1)^2(k+2) = 0$ (ft their factorisation) **M1**

(c) k = 1 or k = -2 cao, both values stated **A1**

**(c) Answer: k = 1 or k = -2**

## Question 5

(a) Q = [[2,3,1],[1,2,2],[3,1,1]] and r = $(34,24,32)^T$ both correct oe **B1**

**(a) Answer: Q = [[2,3,1],[1,2,2],[3,1,1]], r = $(34,24,32)^T$**

(b) correct expansion along a row or column, ft their Q **M1**

(b) correct simplification **A1**

(b) det(Q) = 10 cao **A1**

**(b) Answer: det(Q) = 10**

(c) attempt at cofactor matrix of Q, ft their Q, det(Q) **M1**

(c) correct adjugate (transpose of cofactor matrix) **M1**

(c) $Q^{-1}$ = (1/10)[[0,-2,4],[5,-1,-3],[-5,7,1]] with at least 6 entries correct **A1**

(c) fully correct $Q^{-1}$ cao **A1**

**(c) Answer: $Q^{-1}$ = (1/10)[[0,-2,4],[5,-1,-3],[-5,7,1]]**

(d) computing v = $Q^{-1}$*r, ft their $Q^{-1}$ **M1**

(d) a = 8, c = 5 both correct **A1**

(d) s = 3 cao, all three values stated with correct units (pounds) **A1**

**(d) Answer: Adult ticket = £8, child ticket = £5, senior ticket = £3**


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## Question 6

(a) forming det(W-λ*I) = (7-λ)(-2-λ) - (-6)(3) **M1**

(a) correct quadratic $\lambda^2$ - 5*λ + 4 = 0 oe **A1**

(a) correct factorisation (λ-1)(λ-4) = 0 **M1**

(a) λ = 1, λ = 4, both cao **A1**

**(a) Answer: λ = 1 or λ = 4**

(b) forming the image of (1,m) under W: (7-6m, 3-2m) **M1**

(b) requiring the image to lie on y=mx: 3-2m = m(7-6m) **M1**

(b) forming the quadratic $6m^2$ - 9m + 3 = 0 oe, i.e. $2m^2$-3m+1=0 **M1**

(b) correct factorisation (2m-1)(m-1) = 0 **M1**

(b) both invariant lines stated: y = x and y = (1/2)x cao **A1**

**(b) Answer: y = x and y = x/2**

(c) y = x, ft their lines **B1**

(c) correct justification: this line corresponds to eigenvalue 1, so every point on it maps to itself (scale factor 1), e.g. W(1,1)=(1,1) **B1**

**(c) Answer: y = x consists entirely of invariant points, since it corresponds to eigenvalue λ = 1.**


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