1.3 Further Algebra and Roots: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The cubic equation $2x^3 + 3x^2 - 5x + 1 = 0$ has roots α, β and γ.

(a) State the value of α+β+γ, the value of α*β+β*γ+γ*α, and the value of α*β*γ. **(3)**

(b) Find, in the form $y^3 + by^2 + cy + d = 0$ with integer coefficients, the equation with roots 1/α, 1/β and 1/γ. **(5)**

(c) Show that 1/α + 1/β + 1/γ = 5. **(2)**

(Total for Question 1 is 10 marks)

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2 The quartic equation $x^4 - 2x^3 - 7x^2 + 8x + 12 = 0$ has roots α, β, γ and delta.

(a) State the value of each of: α+β+γ+delta; the sum of the products of the roots taken two at a time; the sum of the products of the roots taken three at a time; α*β*γ*delta. **(4)**

(b) Find the value of $\alpha^2+\beta^2+\gamma^2+\text{delta}^2$. **(4)**

(c) Find the value of 1/α + 1/β + 1/γ + 1/delta. **(3)**

(Total for Question 2 is 11 marks)

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3 The quadratic equation $2x^2 + (k-3)x + (k+1) = 0$, where k is a constant, has roots α and β such that α = 2*β.

(a) Write down expressions, in terms of k, for α + β and for α*β. **(2)**

(b) Using α = 2*β, show that $k^2 - 15k = 0$. **(5)**

(c) Hence find the possible value(s) of k. **(2)**

(Total for Question 3 is 9 marks)

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4 The cubic equation $z^3 + pz^2 + qz + 24 = 0$, where p and q are real constants, has a root -2 + 2i.

(a) Write down another complex root of the equation, with a reason. **(2)**

(b) Find the third root of the equation. **(3)**

(c) Find the values of p and q. **(4)**

(Total for Question 4 is 9 marks)

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5 Solve the inequality $(2x - 4)/(x - 1) > -1$.

(a) Show that the inequality is equivalent to $(3x - 5)/(x - 1) > 0$. **(3)**

(b) Hence solve the inequality. **(3)**

(Total for Question 5 is 6 marks)

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6 The roots of the cubic equation $x^3 + 2x^2 - 21x + 18 = 0$ are α, β and γ.

(a) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(3)**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$, and of $1/\alpha + 1/\beta + 1/\gamma$. **(4)**

(c) Find a cubic equation with integer coefficients whose roots are α, β, γ. **(3)**

(Total for Question 6 is 10 marks)

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Mark scheme · 1.3 Further Algebra and Roots: 1-hour Pack

Question 1

(a) $\alpha+\beta+\gamma = -3/2$ oe **B1**

(a) $\alpha*\beta+\beta*\gamma+\gamma*\alpha = -5/2$ oe **B1**

(a) $\alpha*\beta*\gamma = -1/2$ oe **B1**

(a) Answer: sum = -3/2, sum of pairwise products = -5/2, product = -1/2

(b) letting $y = 1/x$ so $x = 1/y$ **M1**

(b) substituting $x = 1/y$ into $2x^3+3x^2-5x+1=0$ **M1**

(b) multiplying through by y^3 to clear fractions **dM1**

(b) obtaining $2 + 3y - 5y^2 + y^3 = 0$ **A1**

(b) $y^3 - 5y^2 + 3y + 2 = 0$ cao **A1**

(b) Answer: $y^3 - 5y^2 + 3y + 2 = 0$

(c) using $1/\alpha+1/\beta+1/\gamma = (\alpha*\beta+\beta*\gamma+\gamma*\alpha)/(\alpha*\beta*\gamma)$ **M1**

(c) cso: $(-5/2)/(-1/2) = 5$ **A1**

(c) Answer: $1/\alpha + 1/\beta + 1/\gamma = 5$

Question 2

(a) sum = 2 **B1**

(a) sum of pairwise products = -7 **B1**

(a) sum of triple products = -8 **B1**

(a) product = 12 **B1**

(a) Answer: sum = 2, pairwise sum = -7, triple sum = -8, product = 12

(b) using sum of squares = $(\text{sum})^2$ - 2(sum of pairwise products) **M1**

(b) substituting $2^2 - 2(-7)$ **M1**

(b) = 4 + 14 **A1**

(b) = 18 cao **A1**

(b) Answer: $\alpha^2+\beta^2+\gamma^2+\text{delta}^2 = 18$

(c) using sum of reciprocals = (sum of triple products)/(product of all roots) **M1**

(c) = -8/12 **A1**

(c) = -2/3 oe cao **A1**

(c) Answer: $1/\alpha+1/\beta+1/\gamma+1/\text{delta} = -2/3$

Question 3

(a) $\alpha+\beta = -(k-3)/2$ oe **B1**

(a) $\alpha*\beta = (k+1)/2$ oe **B1**

(a) Answer: $\alpha+\beta = (3-k)/2$, $\alpha*\beta = (k+1)/2$

(b) writing $\alpha+\beta = 3*\beta = (3-k)/2$, so $\beta = (3-k)/6$ **M1**

(b) writing $\alpha*\beta = 2*\beta^2 = (k+1)/2$, so $\beta^2 = (k+1)/4$ **M1**

(b) substituting $\beta = (3-k)/6$ into $\beta^2 = (k+1)/4$: $(3-k)^2/36 = (k+1)/4$ **M1**

(b) multiplying through by 36 and expanding: $(3-k)^2 = 9(k+1)$ **M1**

(b) cso: expanding and simplifying $9-6k+k^2 = 9k+9$ to give $k^2-15k=0$ **A1**

(b) Answer: $k^2 - 15k = 0$

(c) factorising $k(k-15) = 0$ **M1**

(c) k = 0 or k = 15, both values stated **A1**

(c) Answer: k = 0 or k = 15

Question 4

(a) -2 - 2i **B1**

(a) complex roots of a polynomial with real coefficients occur in conjugate pairs **B1**

(a) Answer: -2 - 2i

(b) uses the product of the roots: $(\alpha)(\alpha*)(r) = -(24)$ **M1**

(b) $((-2)^2 + 2^2)\, r = -24$ **A1**

(b) r = -3 **A1**

(b) Answer: -3

(c) uses sum of roots = -p **M1**

(c) p = 7 **A1**

(c) uses the sum of products of pairs = q **M1**

(c) q = 20 **A1**

(c) Answer: p = 7, q = 20

Question 5

(a) subtracts -1 from both sides **M1**

(a) puts over a common denominator **M1**

(a) obtains the printed form (cso) **A1**

(a) Answer: $(3x - 5)/(x - 1) > 0$

(b) critical values x = 5/3 and x = 1 (asymptote) **B1**

(b) considers the sign of numerator and denominator in each region (or multiplies by the square of the denominator) **M1**

(b) $x < 1$ or $x > 5/3$ **A1**

(b) Answer: $x < 1$ or $x > 5/3$

Question 6

(a) $\alpha + \beta + \gamma = -2$ **B1**

(a) $\alpha\beta + \beta\gamma + \gamma\alpha = -21$ **B1**

(a) $\alpha\beta\gamma = -18$ **B1**

(a) Answer: -2, -21, -18

(b) $(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ **M1**

(b) 46 **A1**

(b) $(\alpha\beta + \beta\gamma + \gamma\alpha)/(\alpha\beta\gamma)$ **M1**

(b) 7/6 **A1**

(b) Answer: 46; 7/6

(c) substitutes y = x, so x = y **M1**

(c) expands and simplifies **M1**

(c) $y^3 + 2y^2 - 21y + 18 = 0$ **A1**

(c) Answer: $y^3 + 2y^2 - 21y + 18 = 0$

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