1.3 Further Algebra and Roots: 30-minute Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ has roots α, β and γ.

(a) State the value of α+β+γ, the value of α*β + β*γ + γ*α, and the value of α*β*γ. **(3)**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **(3)**

(c) Find the value of 1/α + 1/β + 1/γ, and the value of 1/(α*β) + 1/(β*γ) + 1/(γ*α). **(4)**

(Total for Question 1 is 10 marks)

---

2 The cubic equation $x^3 + 4x^2 - 3x + 7 = 0$ has roots α, β and γ.

(a) State the value of α+β+γ, the value of α*β+β*γ+γ*α, and the value of α*β*γ. **(3)**

(b) Find, in the form $y^3 + by^2 + cy + d = 0$, the equation whose roots are -α, -β and -γ. **(4)**

(c) Explain how the equation with roots -α, -β, -γ can be written down directly from $x^3+4x^2-3x+7=0$, without carrying out a full substitution. **(2)**

(Total for Question 2 is 9 marks)

---

3 The quartic equation $x^4 - 8x^3 + 18x^2 - 10x + 3 = 0$ has roots that are to be examined using the substitution x = y + 2.

(a) Explain why the substitution x = y + 2, rather than some other shift x = y + k, removes the term in y^3 from the resulting quartic in y. **(2)**

(b) Show that the substitution x = y + 2 transforms $x^4 - 8x^3 + 18x^2 - 10x + 3 = 0$ into $y^4 - 6y^2 - 2y + 7 = 0$. **(6)**

(Total for Question 3 is 8 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.3 Further Algebra and Roots: 30-minute Pack

Question 1

(a) $\alpha+\beta+\gamma = 6$ **B1**

(a) $\alpha*\beta+\beta*\gamma+\gamma*\alpha = 11$ **B1**

(a) $\alpha*\beta*\gamma = 6$ **B1**

(a) Answer: sum = 6, sum of pairwise products = 11, product = 6

(b) using sum of squares = $(\text{sum})^2$ - 2(sum of pairwise products) **M1**

(b) substituting 6^2 - 2(11) **M1**

(b) $\alpha^2+\beta^2+\gamma^2 = 14$ cao **A1**

(b) Answer: $\alpha^2+\beta^2+\gamma^2 = 14$

(c) using $1/\alpha+1/\beta+1/\gamma$ = (sum of pairwise products)/(product) **M1**

(c) = 11/6 oe cao **A1**

(c) using 1/(ab)+1/(bg)+1/(ga) = (sum of roots)/(product) **M1**

(c) = 1 cao **A1**

(c) Answer: $1/\alpha+1/\beta+1/\gamma = 11/6$; 1/(ab)+1/(bg)+1/(ga) = 1

Question 2

(a) $\alpha+\beta+\gamma = -4$ **B1**

(a) $\alpha*\beta+\beta*\gamma+\gamma*\alpha = -3$ **B1**

(a) $\alpha*\beta*\gamma = -7$ **B1**

(a) Answer: sum = -4, sum of pairwise products = -3, product = -7

(b) letting y = -x so x = -y **M1**

(b) substituting x = -y into $x^3+4x^2-3x+7=0$ **M1**

(b) obtaining $-y^3+4y^2+3y+7 = 0$ **A1**

(b) $y^3 - 4y^2 - 3y - 7 = 0$ cao **A1**

(b) Answer: $y^3 - 4y^2 - 3y - 7 = 0$

(c) reversing the sign of every other coefficient, starting with the x^2 term (or the constant and x^1 term) oe **B1**

(c) correctly identifying which terms change sign, e.g. the x^3 and x coefficients keep their sign, the x^2 and constant terms change sign oe, ft consistent with part (b) **B1**

(c) Answer: Replace x by -x: this reverses the sign of the coefficients of the even-degree terms below the leading term (here x^2 and the constant), leaving the x^3 and x coefficients' signs unchanged, giving $y^3-4y^2-3y-7=0$.

Question 3

(a) the sum of the roots of the original quartic is 8, so shifting each root by -2 makes the new roots sum to zero oe **B1**

(a) a sum of zero for the new roots is equivalent to the y^3 coefficient of the new quartic being zero oe **B1**

(a) Answer: The four roots of the original equation sum to 8 (since -(-8)/1=8). Shifting each root down by 2 gives new roots summing to 0, which is equivalent to the coefficient of y^3 in the new quartic being 0.

(b) expanding $(y+2)^2 = y^2+4y+4$ **M1**

(b) expanding $(y+2)^3 = y^3+6y^2+12y+8$ **M1**

(b) expanding $(y+2)^4 = y^4+8y^3+24y^2+32y+16$ **M1**

(b) substituting all four expansions into the equation and combining like terms **M1**

(b) y^3 terms cancel (8-8=0) and correct y^2, y coefficients found (-6, -2) **A1**

(b) cso: $y^4 - 6y^2 - 2y + 7 = 0$ **A1**

(b) Answer: $y^4 - 6y^2 - 2y + 7 = 0$

© Revision Library | revisionlibrary.co.uk