1.3 Further Algebra and Roots: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The quadratic equation $x^2 - 6x + 4 = 0$ has roots α and β.

(a) Write down the value of $\alpha + \beta$. **(1)**

(b) Write down the value of $\alpha * \beta$. **(1)**

(c) Find the value of $\alpha^2 + \beta^2$. **(3)**

(d) Which of the following is the quadratic equation, with integer coefficients, whose roots are $\alpha + 2$ and $\beta + 2$? **(2)**

- ☐ A) $y^2 - 10y + 20 = 0$
- ☐ B) $y^2 + 10y + 20 = 0$
- ☐ C) $y^2 - 10y - 20 = 0$
- ☐ D) $y^2 - 2y + 4 = 0$

(Total for Question 1 is 7 marks)

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2 The cubic equation $x^3 - 3x^2 + 5x - 4 = 0$ has roots α, β and γ.

(a) State the value of $\alpha+\beta+\gamma$, the value of $\alpha*\beta+\beta*\gamma+\gamma*\alpha$, and the value of $\alpha*\beta*\gamma$. **(2)**

(b) Show that the cubic equation with roots α^2, β^2 and γ^2 is $y^3 + y^2 + y - 16 = 0$. **(6)**

(Total for Question 2 is 8 marks)

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3 Solve the inequality $(4x + 9)/(x - 2) > 1$.

(a) Show that the inequality is equivalent to $(3x + 11)/(x - 2) > 0$. **(3)**

(b) Hence solve the inequality. **(3)**

(Total for Question 3 is 6 marks)

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4 The roots of the equation $x^2 - 7 = 0$ are α and β.

(a) Write down $\alpha + \beta$ and $\alpha\,\beta$. **(2)**

(b) Find the values of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$. **(4)**

(c) Find a quadratic equation with integer coefficients whose roots are α^2 and β^2. **(3)**

(Total for Question 4 is 9 marks)

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5 The cubic equation $z^3 + pz^2 + qz + 25 = 0$, where p and q are real constants, has a root $3 + 4i$.

(a) Write down another complex root of the equation, with a reason. **(2)**

(b) Find the third root of the equation. **(3)**

(c) Find the values of p and q. **(4)**

(Total for Question 5 is 9 marks)

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6 The quartic equation $x^4 - 15x^3 + 70x^2 - 120x + k = 0$, where k is a constant, has four roots that form a geometric progression a, ar, ar^2, ar^3 (with $r > 1$).

(a) State, in terms of a and r, expressions for $m = a*(ar^3)$ (the product of the outer pair of roots), $S1 = a + ar^3$ (the sum of the outer pair), and $S2 = ar + ar^2$ (the sum of the inner pair). Explain why the product of the inner pair, $(ar)(ar^2)$, is also equal to m. **(2)**

(b) By writing the quartic as $(x^2 - S1*x + m)(x^2 - S2*x + m)$, find the value of k and determine the four roots of the equation. **(8)**

(Total for Question 6 is 10 marks)

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Mark scheme · 1.3 Further Algebra and Roots: Test Pack

Question 1

(a) $\alpha + \beta = 6$ oe **B1**

(a) Answer: $\alpha + \beta = 6$

(b) $\alpha*\beta = 4$ cao **B1**

(b) Answer: $\alpha*\beta = 4$

(c) using $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2*\alpha*\beta$ **M1**

(c) substituting $6^2 - 2(4)$ **M1**

(c) $\alpha^2 + \beta^2 = 28$ cao **A1**

(c) Answer: $\alpha^2 + \beta^2 = 28$

(d) finding new sum = 6+4 = 10 and/or new product = 4+2(6)+4 = 20 **M1**

(d) A cao **A1**

(d) Answer: A) $y^2 - 10y + 20 = 0$

Question 2

(a) sum = 3 and sum of pairwise products = 5 **B1**

(a) product = 4 **B1**

(a) Answer: sum = 3, sum of pairwise products = 5, product = 4

(b) using new sum = $(\alpha+\beta+\gamma)^2 - 2(\alpha*\beta+\beta*\gamma+\gamma*\alpha)$ **M1**

(b) new sum = 9 - 10 = -1 **A1**

(b) using new pairwise sum = $(\alpha*\beta+\beta*\gamma+\gamma*\alpha)^2 - 2*(\alpha*\beta*\gamma)*(\alpha+\beta+\gamma)$ **M1**

(b) new pairwise sum = 25 - 24 = 1 **A1**

(b) new product = $(\alpha*\beta*\gamma)^2 = 16$ **M1**

(b) cso: $y^3 - (-1)y^2 + (1)y - 16 = 0$, i.e. $y^3+y^2+y-16=0$ **A1**

(b) Answer: $y^3 + y^2 + y - 16 = 0$

Question 3

(a) subtracts 1 from both sides **M1**

(a) puts over a common denominator **M1**

(a) obtains the printed form (cso) **A1**

(a) Answer: $(3x + 11)/(x - 2) > 0$

(b) critical values x = -11/3 and x = 2 (asymptote) **B1**

(b) considers the sign of numerator and denominator in each region (or multiplies by the square of the denominator) **M1**

(b) $x < -11/3$ or $x > 2$ **A1**

(b) Answer: $x < -11/3$ or $x > 2$

Question 4

(a) $\alpha + \beta = 0$ **B1**

(a) $\alpha\,\beta = -7$ **B1**

(a) Answer: $\alpha + \beta = 0$, $\alpha\,\beta = -7$

(b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\,\alpha\,\beta$ **M1**

(b) 14 **A1**

(b) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\,\alpha\,\beta\,(\alpha + \beta)$ **M1**

(b) 0 **A1**

(b) Answer: 14; 0

(c) sum of new roots = $\alpha^2 + \beta^2$ and product = $(\alpha\,\beta)^2$ **M1**

(c) sum 14, product 49 **A1**

(c) $y^2 - 14y + 49 = 0$ **A1**

(c) Answer: $y^2 - 14y + 49 = 0$

Question 5

(a) 3 - 4i **B1**

(a) complex roots of a polynomial with real coefficients occur in conjugate pairs **B1**

(a) Answer: 3 - 4i

(b) uses the product of the roots: $(\alpha)(\alpha^*)(r) = -(25)$ **M1**

(b) $((3)^2 + 4^2)\,r = -25$ **A1**

(b) r = -1 **A1**

(b) Answer: -1

(c) uses sum of roots = -p **M1**

(c) p = -5 **A1**

(c) uses the sum of products of pairs = q **M1**

(c) q = 19 **A1**

(c) Answer: p = -5, q = 19

Question 6

(a) $m = a^2*r^3$, $S1 = a+ar^3$, $S2 = ar+ar^2$ all stated correctly **B1**

(a) $(ar)(ar^2) = a^2*r^3 = m$, so the outer and inner pairs have equal products oe **B1**

(a) Answer: $m = a^2\,r^3$, $S1 = a + ar^3$, $S2 = ar + ar^2$; outer product = inner product = $a^2\,r^3 = m$.

(b) expanding $(x^2-S1x+m)(x^2-S2x+m) = x^4-(S1+S2)x^3+(2m+S1*S2)x^2-m(S1+S2)x+m^2$ **M1**

(b) comparing x^3 coefficient: S1+S2 = 15 **M1**

(b) comparing x coefficient: m(S1+S2) = 120, so m = 120/15 **M1**

(b) m = 8 **A1**

(b) $k = m^2 = 64$ (comparing constant terms) **A1**

(b) comparing x^2 coefficient: 2m+S1*S2 = 70, so S1*S2 = 54; solving $t^2-15t+54=0$ to get S1, S2 = 6, 9 **M1**

(b) solving $t^2-9t+8=0$ (outer pair) and $t^2-6t+8=0$ (inner pair) **M1**

(b) roots are 1, 2, 4, 8 (common ratio r=2), with k=64 **A1**

(b) Answer: k = 64; the four roots are 1, 2, 4 and 8.

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