1.3 Further Algebra and Roots

EDEXCEL 9FM0 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The quadratic equation $3x^2 - 7x + 2 = 0$ has roots α and β.

(a) State the value of $\alpha + \beta$ and the value of $\alpha*\beta$. **(2)**

(b) Find the value of $\alpha^2 + \beta^2$. **(3)**

(c) Find, in the form $ay^2 + by + c = 0$ with integer coefficients, the equation whose roots are $2*\alpha + 1$ and $2*\beta + 1$. **(4)**

(Total for Question 1 is 9 marks)

2 The cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ has roots α, β and γ.

(a) State the value of $\alpha+\beta+\gamma$, the value of $\alpha*\beta + \beta*\gamma + \gamma*\alpha$, and the value of $\alpha*\beta*\gamma$. **(3)**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **(3)**

(c) Find the value of $1/\alpha + 1/\beta + 1/\gamma$, and the value of $1/(\alpha*\beta) + 1/(\beta*\gamma) + 1/(\gamma*\alpha)$. **(4)**

(Total for Question 2 is 10 marks)

3 The cubic equation $x^3 + 4x^2 - 3x + 7 = 0$ has roots α, β and γ.

(a) State the value of α+β+γ, the value of α*β+β*γ+γ*α, and the value of α*β*γ. **(3)**

(b) Find, in the form $y^3 + by^2 + cy + d = 0$, the equation whose roots are -α, -β and -γ. **(4)**

(c) Explain how the equation with roots -α, -β, -γ can be written down directly from $x^3+4x^2-3x+7=0$, without carrying out a full substitution. **(2)**

(Total for Question 3 is 9 marks)

4 The cubic equation $2x^3 + 3x^2 - 5x + 1 = 0$ has roots α, β and γ.

(a) State the value of α+β+γ, the value of α*β+β*γ+γ*α, and the value of α*β*γ. **(3)**

(b) Find, in the form $y^3 + by^2 + cy + d = 0$ with integer coefficients, the equation with roots 1/α, 1/β and 1/γ. **(5)**

(c) Show that $1/\alpha + 1/\beta + 1/\gamma = 5$. **(2)**

(Total for Question 4 is 10 marks)

5 The quartic equation $x^4 - 2x^3 - 7x^2 + 8x + 12 = 0$ has roots α, β, γ and delta.

(a) State the value of each of: α+β+γ+delta; the sum of the products of the roots taken two at a time; the sum of the products of the roots taken three at a time; α*β*γ*delta. **(4)**

(b) Find the value of $\alpha^2+\beta^2+\gamma^2+\text{delta}^2$. **(4)**

(c) Find the value of 1/α + 1/β + 1/γ + 1/delta. **(3)**

(Total for Question 5 is 11 marks)

6 The quartic equation $x^4 - 8x^3 + 18x^2 - 10x + 3 = 0$ has roots that are to be examined using the substitution $x = y + 2$.

(a) Explain why the substitution $x = y + 2$, rather than some other shift $x = y + k$, removes the term in y^3 from the resulting quartic in y. **(2)**

(b) Show that the substitution $x = y + 2$ transforms $x^4 - 8x^3 + 18x^2 - 10x + 3 = 0$ into $y^4 - 6y^2 - 2y + 7 = 0$. **(6)**

(Total for Question 6 is 8 marks)

7 The quadratic equation $2x^2 + (k-3)x + (k+1) = 0$, where k is a constant, has roots α and β such that $\alpha = 2*\beta$.

(a) Write down expressions, in terms of k, for $\alpha + \beta$ and for $\alpha*\beta$. **(2)**

(b) Using $\alpha = 2*\beta$, show that $k^2 - 15k = 0$. **(5)**

(c) Hence find the possible value(s) of k. **(2)**

(Total for Question 7 is 9 marks)

8 The cubic equation $z^3 + pz^2 + qz + 24 = 0$, where p and q are real constants, has a root $-2 + 2i$.

(a) Write down another complex root of the equation, with a reason. **(2)**

(b) Find the third root of the equation. **(3)**

(c) Find the values of p and q. **(4)**

(Total for Question 8 is 9 marks)

9 Solve the inequality $(2x - 4)/(x - 1) > -1$.

(a) Show that the inequality is equivalent to $(3x - 5)/(x - 1) > 0$. **(3)**

(b) Hence solve the inequality. **(3)**

(Total for Question 9 is 6 marks)

10 The roots of the cubic equation $x^3 + 2x^2 - 21x + 18 = 0$ are α, β and γ.

(a) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(3)**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$, and of $1/\alpha + 1/\beta + 1/\gamma$. **(4)**

(c) Find a cubic equation with integer coefficients whose roots are α, β, γ. **(3)**

(Total for Question 10 is 10 marks)

11 The roots of the cubic equation $x^3 + 7x^2 + 14x + 8 = 0$ are α, β and γ.

(a) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(3)**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$, and of $1/\alpha + 1/\beta + 1/\gamma$. **(4)**

(c) Find a cubic equation with integer coefficients whose roots are $\alpha + 2$, $\beta + 2$, $\gamma + 2$. **(3)**

(Total for Question 11 is 10 marks)

12 The roots of the cubic equation $x^3 - 7x^2 + 14x - 8 = 0$ are α, β and γ.

(a) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(3)**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$, and of $1/\alpha + 1/\beta + 1/\gamma$. **(4)**

(c) Find a cubic equation with integer coefficients whose roots are $\alpha + 3$, $\beta + 3$, $\gamma + 3$. **(3)**

(Total for Question 12 is 10 marks)

13 The roots of the equation $x^2 - 8x - 8 = 0$ are α and β.

(a) Write down $\alpha + \beta$ and $\alpha\beta$. **(2)**

(b) Find the values of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$. **(4)**

(c) Find a quadratic equation with integer coefficients whose roots are α^2 and β^2. **(3)**

(Total for Question 13 is 9 marks)

14 The roots of the equation $x^2 + 6x + 8 = 0$ are α and β.

(a) Write down $\alpha + \beta$ and $\alpha\beta$. **(2)**

(b) Find the values of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$. **(4)**

(c) Find a quadratic equation with integer coefficients whose roots are α^2 and β^2. **(3)**

(Total for Question 14 is 9 marks)

15 The roots of the equation $x^2 + 4x - 1 = 0$ are α and β.

(a) Write down $\alpha + \beta$ and $\alpha\beta$. **(2)**

(b) Find the values of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$. **(4)**

(c) Find a quadratic equation with integer coefficients whose roots are α^2 and β^2. **(3)**

(Total for Question 15 is 9 marks)

16 The roots of the cubic equation $x^3 + 13x^2 + 52x + 60 = 0$ are α, β and γ.

(a) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(3)**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$, and of $1/\alpha + 1/\beta + 1/\gamma$. **(4)**

(c) Find a cubic equation with integer coefficients whose roots are α, β, γ. **(3)**

(Total for Question 16 is 10 marks)

17 The cubic equation $x^3 - 12x^2 + kx - 28 = 0$, where k is a constant, has three roots that are in arithmetic progression.

(a) By writing the three roots as (a-d), a and (a+d), explain why a = 4 and hence why x = 4 must satisfy the equation. **(2)**

(b) Find the value of k. **(3)**

(c) Hence find the other two roots of the equation. **(4)**

(Total for Question 17 is 9 marks)

18 Given that $2+i$ is a root of $x^3 - 7x^2 + 17x - 15 = 0$, and that the equation has real coefficients.

(a) State, with a reason, another root of the equation. **(2)**

(b) Find the third root of the equation. **(4)**

(c) Hence express $x^3 - 7x^2 + 17x - 15$ as the product of a linear factor and a quadratic factor, both with real coefficients. **(3)**

(Total for Question 18 is 9 marks)

19 A cubic equation has roots α, β and γ, with $\alpha+\beta+\gamma = p$, $\alpha*\beta+\beta*\gamma+\gamma*\alpha = q$ and $\alpha*\beta*\gamma = r$.

(a) Prove that $\alpha^3 + \beta^3 + \gamma^3 = p^3 - 3pq + 3r$. **(5)**

(b) Hence find the value of $\alpha^3 + \beta^3 + \gamma^3$ for the equation $x^3 - 5x^2 + 6x - 2 = 0$. **(3)**

(Total for Question 19 is 8 marks)

Mark scheme · 1.3 Further Algebra and Roots

Question 1

(a) $\alpha + \beta = 7/3$ oe **B1**

(a) $\alpha*\beta = 2/3$ oe **B1**

(a) Answer: $\alpha + \beta = 7/3$, $\alpha*\beta = 2/3$

(b) using $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2*\alpha*\beta$ **M1**

(b) correct substitution $(7/3)^2 - 2(2/3)$ **A1**

(b) $\alpha^2+\beta^2 = 37/9$ oe cao **A1**

(b) Answer: $\alpha^2 + \beta^2 = 37/9$

(c) letting $y = 2x+1$ so $x = (y-1)/2$ **M1**

(c) substituting $x = (y-1)/2$ into $3x^2-7x+2=0$ **M1**

(c) multiplying through by 4 and collecting terms **dM1**

(c) $3y^2 - 20y + 25 = 0$ cao **A1**

(c) Answer: $3y^2 - 20y + 25 = 0$

Question 2

(a) $\alpha+\beta+\gamma = 6$ **B1**

(a) $\alpha*\beta+\beta*\gamma+\gamma*\alpha = 11$ **B1**

(a) $\alpha*\beta*\gamma = 6$ **B1**

(a) Answer: sum = 6, sum of pairwise products = 11, product = 6

(b) using sum of squares = $(\text{sum})^2$ - 2(sum of pairwise products) **M1**

(b) substituting $6^2 - 2(11)$ **M1**

(b) $\alpha^2+\beta^2+\gamma^2 = 14$ cao **A1**

(b) Answer: $\alpha^2+\beta^2+\gamma^2 = 14$

(c) using $1/\alpha+1/\beta+1/\gamma$ = (sum of pairwise products)/(product) **M1**

(c) = 11/6 oe cao **A1**

(c) using 1/(ab)+1/(bg)+1/(ga) = (sum of roots)/(product) **M1**

(c) = 1 cao **A1**

(c) Answer: $1/\alpha+1/\beta+1/\gamma = 11/6$; 1/(ab)+1/(bg)+1/(ga) = 1

Question 3

(a) $\alpha+\beta+\gamma = -4$ **B1**

(a) $\alpha*\beta+\beta*\gamma+\gamma*\alpha = -3$ **B1**

(a) $\alpha*\beta*\gamma = -7$ **B1**

(a) Answer: sum = -4, sum of pairwise products = -3, product = -7

(b) letting $y = -x$ so $x = -y$ **M1**

(b) substituting $x = -y$ into $x^3+4x^2-3x+7=0$ **M1**

(b) obtaining $-y^3+4y^2+3y+7 = 0$ **A1**

(b) $y^3 - 4y^2 - 3y - 7 = 0$ cao **A1**

(b) Answer: $y^3 - 4y^2 - 3y - 7 = 0$

(c) reversing the sign of every other coefficient, starting with the x^2 term (or the constant and x^1 term) oe **B1**

(c) correctly identifying which terms change sign, e.g. the x^3 and x coefficients keep their sign, the x^2 and constant terms change sign oe, ft consistent with part (b) **B1**

(c) Answer: Replace x by -x: this reverses the sign of the coefficients of the even-degree terms below the leading term (here x^2 and the constant), leaving the x^3 and x coefficients' signs unchanged, giving $y^3-4y^2-3y-7=0$.

Question 4

(a) $\alpha+\beta+\gamma = -3/2$ oe **B1**

(a) $\alpha*\beta+\beta*\gamma+\gamma*\alpha = -5/2$ oe **B1**

(a) $\alpha*\beta*\gamma = -1/2$ oe **B1**

(a) Answer: sum = -3/2, sum of pairwise products = -5/2, product = -1/2

(b) letting $y = 1/x$ so $x = 1/y$ **M1**

(b) substituting $x = 1/y$ into $2x^3+3x^2-5x+1=0$ **M1**

(b) multiplying through by y^3 to clear fractions **dM1**

(b) obtaining $2 + 3y - 5y^2 + y^3 = 0$ **A1**

(b) $y^3 - 5y^2 + 3y + 2 = 0$ cao **A1**

(b) Answer: $y^3 - 5y^2 + 3y + 2 = 0$

(c) using $1/\alpha+1/\beta+1/\gamma = (\alpha*\beta+\beta*\gamma+\gamma*\alpha)/(\alpha*\beta*\gamma)$ **M1**

(c) cso: (-5/2)/(-1/2) = 5 **A1**

(c) Answer: $1/\alpha + 1/\beta + 1/\gamma = 5$

Question 5

(a) sum = 2 **B1**

(a) sum of pairwise products = -7 **B1**

(a) sum of triple products = -8 **B1**

(a) product = 12 **B1**

(a) Answer: sum = 2, pairwise sum = -7, triple sum = -8, product = 12

(b) using sum of squares = $(\text{sum})^2$ - 2(sum of pairwise products) **M1**

(b) substituting $2^2 - 2(-7)$ **M1**

(b) = 4 + 14 **A1**

(b) = 18 cao **A1**

(b) Answer: $\alpha^2+\beta^2+\gamma^2+\text{delta}^2 = 18$

(c) using sum of reciprocals = (sum of triple products)/(product of all roots) **M1**

(c) = -8/12 **A1**

(c) = -2/3 oe cao **A1**

(c) Answer: 1/α+1/β+1/γ+1/delta = -2/3

Question 6

(a) the sum of the roots of the original quartic is 8, so shifting each root by -2 makes the new roots sum to zero oe **B1**

(a) a sum of zero for the new roots is equivalent to the y^3 coefficient of the new quartic being zero oe **B1**

(a) Answer: The four roots of the original equation sum to 8 (since -(-8)/1=8). Shifting each root down by 2 gives new roots summing to 0, which is equivalent to the coefficient of y^3 in the new quartic being 0.

(b) expanding $(y+2)^2 = y^2+4y+4$ **M1**

(b) expanding $(y+2)^3 = y^3+6y^2+12y+8$ **M1**

(b) expanding $(y+2)^4 = y^4+8y^3+24y^2+32y+16$ **M1**

(b) substituting all four expansions into the equation and combining like terms **M1**

(b) y^3 terms cancel (8-8=0) and correct y^2, y coefficients found (-6, -2) **A1**

(b) cso: $y^4 - 6y^2 - 2y + 7 = 0$ **A1**

(b) Answer: $y^4 - 6y^2 - 2y + 7 = 0$

Question 7

(a) $\alpha+\beta = -(k-3)/2$ oe **B1**

(a) $\alpha*\beta = (k+1)/2$ oe **B1**

(a) Answer: $\alpha+\beta = (3-k)/2$, $\alpha*\beta = (k+1)/2$

(b) writing $\alpha+\beta = 3*\beta = (3-k)/2$, so $\beta = (3-k)/6$ **M1**

(b) writing $\alpha*\beta = 2*\beta^2 = (k+1)/2$, so $\beta^2 = (k+1)/4$ **M1**

(b) substituting $\beta = (3-k)/6$ into $\beta^2 = (k+1)/4$: $(3-k)^2/36 = (k+1)/4$ **M1**

(b) multiplying through by 36 and expanding: $(3-k)^2 = 9(k+1)$ **M1**

(b) cso: expanding and simplifying $9-6k+k^2 = 9k+9$ to give $k^2-15k=0$ **A1**

(b) Answer: $k^2 - 15k = 0$

(c) factorising $k(k-15) = 0$ **M1**

(c) $k = 0$ or $k = 15$, both values stated **A1**

(c) Answer: $k = 0$ or $k = 15$

Question 8

(a) $-2 - 2i$ **B1**

(a) complex roots of a polynomial with real coefficients occur in conjugate pairs **B1**

(a) Answer: $-2 - 2i$

(b) uses the product of the roots: $(\alpha)(\alpha^*)(r) = -(24)$ **M1**

(b) $((-2)^2 + 2^2)\, r = -24$ **A1**

(b) $r = -3$ **A1**

(b) Answer: -3

(c) uses sum of roots $= -p$ **M1**

(c) $p = 7$ **A1**

(c) uses the sum of products of pairs $= q$ **M1**

(c) $q = 20$ **A1**

(c) Answer: $p = 7$, $q = 20$

Question 9

(a) subtracts -1 from both sides **M1**

(a) puts over a common denominator **M1**

(a) obtains the printed form (cso) **A1**

(a) Answer: $(3x - 5)/(x - 1) > 0$

(b) critical values $x = 5/3$ and $x = 1$ (asymptote) **B1**

(b) considers the sign of numerator and denominator in each region (or multiplies by the square of the denominator) **M1**

(b) $x < 1$ or $x > 5/3$ **A1**

(b) Answer: $x < 1$ or $x > 5/3$

Question 10

(a) $\alpha + \beta + \gamma = -2$ **B1**

(a) $\alpha\beta + \beta\gamma + \gamma\alpha = -21$ **B1**

(a) $\alpha\beta\gamma = -18$ **B1**

(a) Answer: -2, -21, -18

(b) $(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ **M1**

(b) 46 **A1**

(b) $(\alpha\beta + \beta\gamma + \gamma\alpha)/(\alpha\beta\gamma)$ **M1**

(b) 7/6 **A1**

(b) Answer: 46; 7/6

(c) substitutes $y = x$, so $x = y$ **M1**

(c) expands and simplifies **M1**

(c) $y^3 + 2y^2 - 21y + 18 = 0$ **A1**

(c) Answer: $y^3 + 2y^2 - 21y + 18 = 0$

Question 11

(a) $\alpha + \beta + \gamma = -7$ **B1**

(a) $\alpha\beta + \beta\gamma + \gamma\alpha = 14$ **B1**

(a) $\alpha\beta\gamma = -8$ **B1**

(a) Answer: -7, 14, -8

(b) $(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ **M1**

(b) 21 **A1**

(b) $(\alpha\beta + \beta\gamma + \gamma\alpha)/(\alpha\beta\gamma)$ **M1**

(b) -7/4 **A1**

(b) Answer: 21; -7/4

(c) substitutes $y = x + 2$, so $x = y - 2$ **M1**

(c) expands and simplifies **M1**

(c) $y^3 + y^2 - 2y = 0$ **A1**

(c) Answer: $y^3 + y^2 - 2y = 0$

Question 12

(a) $\alpha + \beta + \gamma = 7$ **B1**

(a) $\alpha\beta + \beta\gamma + \gamma\alpha = 14$ **B1**

(a) $\alpha\beta\gamma = 8$ **B1**

(a) Answer: 7, 14, 8

(b) $(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ **M1**

(b) 21 **A1**

(b) $(\alpha\beta + \beta\gamma + \gamma\alpha)/(\alpha\beta\gamma)$ **M1**

(b) 7/4 **A1**

(b) Answer: 21; 7/4

(c) substitutes $y = x + 3$, so $x = y - 3$ **M1**

(c) expands and simplifies **M1**

(c) $y^3 - 16y^2 + 83y - 140 = 0$ **A1**

(c) Answer: $y^3 - 16y^2 + 83y - 140 = 0$

Question 13

(a) $\alpha + \beta = 8$ **B1**

(a) $\alpha \beta = -8$ **B1**

(a) Answer: $\alpha + \beta = 8$, $\alpha \beta = -8$

(b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2 \alpha \beta$ **M1**

(b) 80 **A1**

(b) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3 \alpha \beta (\alpha + \beta)$ **M1**

(b) 704 **A1**

(b) Answer: 80; 704

(c) sum of new roots = $\alpha^2 + \beta^2$ and product = $(\alpha \beta)^2$ **M1**

(c) sum 80, product 64 **A1**

(c) $y^2 - 80y + 64 = 0$ **A1**

(c) Answer: $y^2 - 80y + 64 = 0$

Question 14

(a) $\alpha + \beta = -6$ **B1**

(a) $\alpha \beta = 8$ **B1**

(a) Answer: $\alpha + \beta = -6$, $\alpha \beta = 8$

(b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2 \alpha \beta$ **M1**

(b) 20 **A1**

(b) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3 \alpha \beta (\alpha + \beta)$ **M1**

(b) -72 **A1**

(b) Answer: 20; -72

(c) sum of new roots = $\alpha^2 + \beta^2$ and product = $(\alpha \beta)^2$ **M1**

(c) sum 20, product 64 **A1**

(c) $y^2 - 20y + 64 = 0$ **A1**

(c) Answer: $y^2 - 20y + 64 = 0$

Question 15

(a) $\alpha + \beta = -4$ **B1**

(a) $\alpha \beta = -1$ **B1**

(a) Answer: $\alpha + \beta = -4$, $\alpha \beta = -1$

(b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2 \alpha \beta$ **M1**

(b) 18 **A1**

(b) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3 \alpha \beta (\alpha + \beta)$ **M1**

(b) -76 **A1**

(b) Answer: 18; -76

(c) sum of new roots = $\alpha^2 + \beta^2$ and product = $(\alpha \beta)^2$ **M1**

(c) sum 18, product 1 **A1**

(c) $y^2 - 18y + 1 = 0$ **A1**

(c) Answer: $y^2 - 18y + 1 = 0$

Question 16

(a) $\alpha + \beta + \gamma = -13$ **B1**

(a) $\alpha \beta + \beta \gamma + \gamma \alpha = 52$ **B1**

(a) $\alpha \beta \gamma = -60$ **B1**

(a) Answer: -13, 52, -60

(b) $(\alpha + \beta + \gamma)^2 - 2(\alpha \beta + \beta \gamma + \gamma \alpha)$ **M1**

(b) 65 **A1**

(b) $(\alpha \beta + \beta \gamma + \gamma \alpha)/(\alpha \beta \gamma)$ **M1**

(b) -13/15 **A1**

(b) Answer: 65; -13/15

(c) substitutes y = x, so x = y **M1**

(c) expands and simplifies **M1**

(c) $y^3 + 13y^2 + 52y + 60 = 0$ **A1**

(c) Answer: $y^3 + 13y^2 + 52y + 60 = 0$

Question 17

(a) sum of roots = (a-d)+a+(a+d) = 3a = 12, so a = 4 **B1**

(a) since a=4 is one of the three roots, x=4 must satisfy the cubic oe **B1**

(a) Answer: a = 4, so x = 4 is a root of the equation.

(b) substituting x=4 into $x^3-12x^2+kx-28=0$ **M1**

(b) 64 - 192 + 4k - 28 = 0 **M1**

(b) k = 39 cao **A1**

(b) Answer: k = 39

(c) dividing $x^3-12x^2+39x-28$ by (x-4) **M1**

(c) obtaining quotient x^2-8x+7 **A1**

(c) factorising (x-1)(x-7) or using the quadratic formula **M1**

(c) x = 1 and x = 7 cao **A1**

(c) Answer: x = 1 and x = 7 (roots 1, 4, 7, common difference 3)

Question 18

(a) 2 - i **B1**

(a) reason: complex roots of a polynomial with real coefficients occur in conjugate pairs oe **B1**

(a) Answer: 2 - i, since non-real roots of a real polynomial occur in conjugate pairs.

(b) using sum of roots = 7, with (2+i)+(2-i) = 4 **M1**

(b) third root = 7 - 4 **M1**

(b) third root = 3 **A1**

(b) verified using product of roots: (2+i)(2-i)*3 = 5*3 = 15, matching -(-15) = 15 ft **A1**

(b) Answer: third root = 3

(c) forming quadratic factor (x-(2+i))(x-(2-i)) **M1**

(c) $x^2 - 4x + 5$ **A1**

(c) $(x-3)(x^2-4x+5)$ cao, ft their roots **A1**

(c) Answer: $x^3 - 7x^2 + 17x - 15 = (x-3)(x^2-4x+5)$

Question 19

(a) quoting or deriving the identity $\alpha^3+\beta^3+\gamma^3-3*\alpha*\beta*\gamma = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha*\beta-\beta*\gamma-\gamma*\alpha)$ **M1**

(a) using $\alpha^2+\beta^2+\gamma^2 = p^2-2q$ **M1**

(a) bracket simplifies to $p^2-2q-q = p^2-3q$ **A1**

(a) so $\alpha^3+\beta^3+\gamma^3 - 3r = p(p^2-3q) = p^3-3pq$ **M1**

(a) cso: $\alpha^3+\beta^3+\gamma^3 = p^3-3pq+3r$ **A1**

(a) Answer: $\alpha^3 + \beta^3 + \gamma^3 = p^3 - 3pq + 3r$

(b) identifying p=5, q=6, r=2 **M1**

(b) substituting into $p^3-3pq+3r$: 125-3(5)(6)+3(2) **M1**

(b) = 41 cao **A1**

(b) Answer: $\alpha^3+\beta^3+\gamma^3 = 41$