1.4 Further Calculus: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question is about improper integrals.

(a) Find the value of the integral from 0 to infinity of $2e^{-2x}$ dx. **(3)**

(Total for Question 1 is 3 marks)

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2 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = e^{4x}$ over the interval $2 \leq x \leq 5$, giving your answer in an exact form. **(3)**

(Total for Question 2 is 3 marks)

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3 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to $4\sqrt{3}/2$ of $1/\sqrt{16 - x^2}$ dx. **(4)**

(Total for Question 3 is 4 marks)

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4 The region R is bounded by the curve $y = x^2 + 2$, the coordinate axes and the line $x = 2$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 4 is 4 marks)

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5 The region R is bounded by the curve $y = \sqrt{5x}$, the x axis and the line $x = 4$. The region R is rotated through 2pi radians about the x axis.

(a) Write down an integral for the volume of the solid formed, and evaluate it exactly. **(4)**

(Total for Question 5 is 4 marks)

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6 The region R is bounded by the curve $y = x^2 + 3$, the coordinate axes and the line $x = 4$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 6 is 4 marks)

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7 This question is about improper integrals.

(a) Use integration by parts to find the exact value of the integral from 0 to infinity of $3x\, e^{-4x}\, dx$. **(4)**

(Total for Question 7 is 4 marks)

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8 This question is about improper integrals.

(a) Find the value of the integral from 0 to infinity of $3e^{-4x}\, dx$. **(3)**

(Total for Question 8 is 3 marks)

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9 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = x^2 + 3x$ over the interval $2 \leq x \leq 3$. **(4)**

(Total for Question 9 is 4 marks)

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10 This question is about improper integrals.

(a) Evaluate the improper integral of $1/\sqrt{x}$ from $x = 0$ to $x = 1$. **(3)**

(Total for Question 10 is 3 marks)

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11 The region R is bounded by the curve $y = \sqrt{6x}$, the x axis and the line $x = 4$. The region R is rotated through 2pi radians about the x axis.

(a) Write down an integral for the volume of the solid formed, and evaluate it exactly. **(4)**

(Total for Question 11 is 4 marks)

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12 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = \sin(4x)$ over the interval $0 \leq x \leq \pi/4$. **(3)**

(Total for Question 12 is 3 marks)

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13 The region R is bounded by the curve $y = 2\sin(3x)$, for $0 \leq x \leq \pi/3$, and the x axis. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 13 is 4 marks)

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Mark scheme · 1.4 Further Calculus: 1-hour Pack

Question 1

(a) integrates: $-(2/2)e^{-2x}$ **M1**

(a) takes the limit as the upper bound tends to infinity **M1**

(a) 1 **A1**

(a) Answer: 1

Question 2

(a) mean value = (1/(b - a)) integral of f **M1**

(a) integrates to $e^{4x}/4$ **M1**

(a) $(e^{20} - e^{8})/12$ **A1**

(a) Answer: $(e^{20} - e^{8})/12$

Question 3

(a) recognises the standard form $1/\sqrt{a^2 - x^2}$ with a = 4 **M1**

(a) integrates: arcsin(x/4) **M1**

(a) $\arcsin(\sqrt{3}/2) - \arcsin(0)$ **A1**

(a) π/3 **A1**

(a) Answer: π/3

Question 4

(a) V = π integral of $(x^2 + 2)^2$ dx **M1**

(a) expands: $x^4 + 4x^2 + 4$ **M1**

(a) integrates term by term and substitutes the limits **M1**

(a) V = 376pi/15 **A1**

(a) Answer: 376pi/15

Question 5

(a) V = π integral of y^2 dx **M1**

(a) π integral from 0 to 4 of 5x dx **A1**

(a) integrates: $\pi[5/2x^2]$ **M1**

(a) V = 40pi **A1**

(a) Answer: 40pi

Question 6

(a) V = π integral of $(x^2 + 3)^2$ dx **M1**

(a) expands: $x^4 + 6x^2 + 9$ **M1**

(a) integrates term by term and substitutes the limits **M1**

(a) V = 1844pi/5 **A1**

(a) Answer: 1844pi/5

Question 7

(a) u = x, dv/dx = e^{-4x} **M1**

(a) $-x\,e^{-4x}/4$ + integral of $e^{-4x}/4$ dx **A1**

(a) uses $x\,e^{-kx}$ tends to 0 as x tends to infinity **M1**

(a) 3/16 **A1**

(a) Answer: 3/16

Question 8

(a) integrates: $-(3/4)e^{-4x}$ **M1**

(a) takes the limit as the upper bound tends to infinity **M1**

(a) 3/4 **A1**

(a) Answer: 3/4

Question 9

(a) mean value = (1/(3 - 2)) integral of f(x) dx **M1**

(a) integrates: $1/3x^3 + 3/2x^2$ **M1**

(a) integral = 83/6 **A1**

(a) mean value = 83/6 **A1**

(a) Answer: 83/6

Question 10

(a) integrates: $2x^{1/2}$ **M1**

(a) replaces the lower limit by t and lets t tend to 0 from above **M1**

(a) 2 **A1**

(a) Answer: 2

Question 11

(a) V = π integral of y^2 dx **M1**

(a) π integral from 0 to 4 of 6x dx **A1**

(a) integrates: $\pi[3x^2]$ **M1**

(a) V = 48pi **A1**

(a) Answer: 48pi

Question 12

(a) mean value = (1/(π/4)) integral of sin(4x) dx **M1**

(a) integral = [-cos(4x)/4] from 0 to π/4 = 2/4 **M1**

(a) mean value = 2/π **A1**

(a) Answer: 2/π

Question 13

(a) V = π integral of y^2 dx **M1**

(a) uses $\sin^2(3x) = (1 - \cos(6x))/2$ **M1**

(a) integrates and substitutes the limits 0 and π/3 **M1**

(a) V = $2pi^2/3$ **A1**

(a) Answer: $2pi^2/3$

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