1.4 Further Calculus: 30-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The region R is bounded by the curve $y = e^{4x}$, the coordinate axes and the line $x = 1$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(3)**

(Total for Question 1 is 3 marks)

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2 This question concerns standard integrals leading to inverse trigonometric and inverse hyperbolic functions.

(a) Find the exact value of the integral: integral of $1/\sqrt{9-x^2}$ dx. **(3)**

(b) Find the integral: integral of $1/\sqrt{x^2+16}$ dx. **(3)**

(c) Hence evaluate integral from x=0 to x=4 of $1/\sqrt{25-x^2}$ dx, giving your answer to 3 significant figures. **(4)**

(Total for Question 2 is 10 marks)

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3 This question is about improper integrals.

(a) Show that the improper integral of $5/x^4$ from x = 1 to infinity converges, and find its value. **(4)**

(Total for Question 3 is 4 marks)

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4 Let I_n = integral from x=0 to x=π/2 of $\sin^n(x)$ dx, for $n \geq 0$.

(a) Using integration by parts on $\sin^n(x) = \sin(x)*\sin^{n-1}(x)$, show that, for $n \geq 2$, $I_n = ((n-1)/n) * I_{n-2}$. **(5)**

(b) State the exact values of I_0 and I_1. **(2)**

(c) Hence find the exact value of I_5, giving your answer as a fraction in its simplest form, and state its value correct to 3 significant figures. **(4)**

(Total for Question 4 is 11 marks)

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Mark scheme · 1.4 Further Calculus: 30-minute Pack

Question 1

(a) V = π integral of e^{8x} dx **M1**

(a) integrates: $\pi[e^{8x}/8]$ **M1**

(a) substitutes the limits: $(\pi/8)(e^8 - 1)$ **A1**

(a) Answer: $(\pi/8)(e^8 - 1)$

Question 2

(a) recognises the standard form integral $1/\sqrt{a^2-x^2}$ dx = arcsin(x/a) + c with a=3 **M1**

(a) arcsin(x/3) seen **A1**

(a) + c included, cao **A1**

(a) Answer: arcsin(x/3) + c

(b) recognises the standard form integral $1/\sqrt{x^2+a^2}$ dx = arsinh(x/a) + c with a=4 **M1**

(b) arsinh(x/4) seen **A1**

(b) + c included, oe (accept equivalent log form $\ln(x+\sqrt{x^2+16})+k$) **A1**

(b) Answer: arsinh(x/4) + c

(c) writes the integral as [arcsin(x/5)] with limits 0 to 4 **M1**

(c) substitutes limits to get arcsin(4/5) - arcsin(0) **M1**

(c) exact value arcsin(4/5) (or arcsin(0.8)), oe **A1**

(c) awrt 0.927 (radians) **A1**

(c) Answer: arcsin(4/5) = awrt 0.927

Question 3

(a) integrates: $5x^{-3}/(-3) = -5/3x^{-3}$ **M1**

(a) replaces the upper limit by t and lets t tend to infinity **M1**

(a) t^{-3} tends to 0 so the integral converges **A1**

(a) value = 5/3 **A1**

(a) Answer: 5/3

Question 4

(a) chooses $u=\sin^{n-1}(x)$, $dv/dx=\sin(x)$, so $du/dx=(n-1)\sin^{n-2}(x)\cos(x)$, $v=-\cos(x)$ **M1**

(a) applies parts and evaluates boundary term $[-\cos(x)\sin^{n-1}(x)]_0^{\pi/2} = 0$ **M1**

(a) uses $\cos^2(x) = 1-\sin^2(x)$ to write (n-1)*integral of $(1-\sin^2(x))\sin^{n-2}(x)$ dx **M1**

(a) expands to $I_n = (n-1)(I_{n-2} - I_n)$ and rearranges to collect I_n terms: $n*I_n = (n-1)*I_{n-2}$ **M1**

(a) $I_n = ((n-1)/n)*I_{n-2}$, cso (printed result) **A1**

(a) Answer: $I_n = ((n-1)/n) * I_{n-2}$ (printed result)

(b) $I_0 = \pi/2$ **B1**

(b) $I_1 = 1$ **B1**

(b) Answer: $I_0 = \pi/2$, $I_1 = 1$

(c) uses $I_3 = (2/3)*I_1 = 2/3$ **M1**

(c) uses $I_5 = (4/5)*I_3$ **M1**

(c) $I_5 = 8/15$, cao **A1**

(c) awrt 0.533 **A1**

(c) Answer: $I_5 = 8/15$ (awrt 0.533)

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