1.4 Further Calculus: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question concerns the differentiation of inverse trigonometric functions.

(a) Given that y = arcsin(4x), find dy/dx, stating the values of x for which the derivative is defined. **(2)**

(b) Given that y = arctan(3x - 1), find dy/dx, and hence find the exact value of dy/dx when x = 1. **(3)**

(c) Show that d/dx[arccos(1 - 2x)] = $1/\sqrt{x - x^2}$, for $0 < x < 1$. **(3)**

(Total for Question 1 is 8 marks)

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2 This question concerns inverse hyperbolic functions.

(a) Given that y = arsinh(x), so that sinh(y) = x, prove that $dy/dx = 1/\sqrt{1+x^2}$. **(4)**

(b) Hence write down dy/dx when y = arsinh(2x). **(2)**

(c) Show that $\text{arsinh}(2x) = \ln(2x + \sqrt{4x^2+1})$. Hence find the exact value of arsinh(4) as a single natural logarithm, and evaluate it correct to 3 significant figures. **(4)**

(Total for Question 2 is 10 marks)

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3 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to $\sqrt{3}$ of $1/(x^2 + 1)$ dx. **(4)**

(Total for Question 3 is 4 marks)

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4 Let I_n = integral from x=0 to x=1 of $x^n * e^x$ dx, for $n \geq 0$.

(a) Using integration by parts, show that, for $n \geq 1$, $I_n = e - n*I_{n-1}$. **(4)**

(b) Write down the exact value of I_0. **(2)**

(c) Hence find the exact value of I_3, giving your answer in the form a - b*e where a and b are integers. **(4)**

(Total for Question 4 is 10 marks)

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5 This question concerns improper integrals.

(a) Show that integral from x=1 to x=R of $1/x^2$ dx = 1 - 1/R. Hence determine whether integral from x=1 to infinity of $1/x^2$ dx converges, and if so find its value. **(3)**

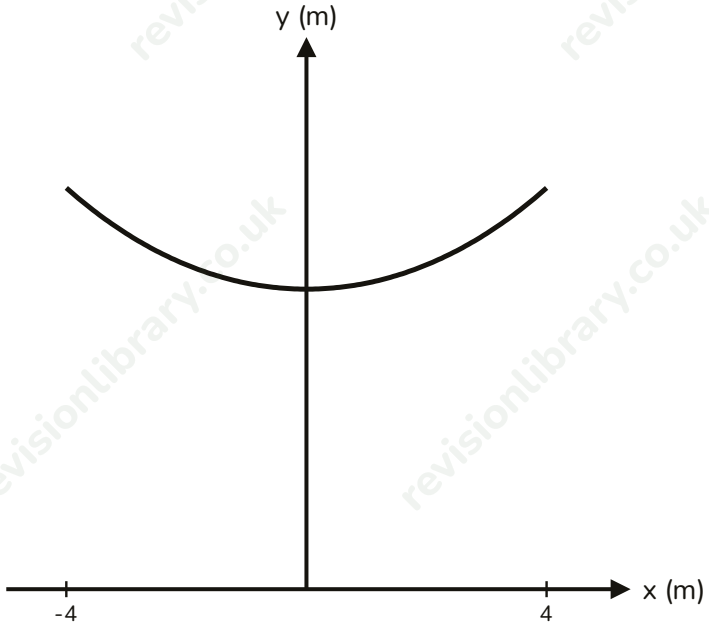
(b) Determine whether integral from x=1 to infinity of $1/\sqrt{x}$ dx converges or diverges, justifying your answer by considering the limit as R tends to infinity. **(4)**

(c) The integral from x=0 to x=1 of $1/\sqrt{x}$ dx is improper due to a singularity at x=0. Show that it converges and find its exact value. **(4)**

(Total for Question 5 is 11 marks)

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6 A structural engineer in Bristol is modelling a section of a suspension cable using the catenary curve y = 5*cosh(x/5), where x and y are measured in metres, for $-4 \leq x \leq 4$.



(a) Find dy/dx, and show that $1 + (dy/dx)^2 = \cosh^2(x/5)$. **(2)**

(b) State the arc length formula, and use the result from part (a) to show that the length of the cable between x = -4 and x = 4 is given by integral from x=-4 to x=4 of cosh(x/5) dx. **(2)**

(c) Hence find the length of this section of cable, giving your answer in metres correct to 3 significant figures. **(4)**

(Total for Question 6 is 8 marks)

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Mark scheme · 1.4 Further Calculus: Test Pack

Question 1

(a) uses d/dx[arcsin(u)] = u'/$\sqrt{1-u^2}$ with u = 4x **M1**

(a) dy/dx = 4/$\sqrt{1-16x^2}$, oe, valid for -1/4 < x < 1/4 **A1**

(a) Answer: dy/dx = 4/$\sqrt{1 - 16x^2}$, for -1/4 < x < 1/4

(b) uses d/dx[arctan(u)] = u'/(1+u^2) with u = 3x-1 **M1**

(b) dy/dx = 3/(1+(3x-1)^2), oe **A1**

(b) substitutes x=1 to obtain dy/dx = 3/5, cao **A1**

(b) Answer: dy/dx = 3/(1+(3x-1)^2); at x = 1, dy/dx = 3/5

(c) uses d/dx[arccos(u)] = -u'/$\sqrt{1-u^2}$ with u = 1-2x, so du/dx = -2 **M1**

(c) 1-(1-2x)^2 simplified to 4x - 4x^2 = 4x(1-x) **M1**

(c) correctly simplifies 2/$\sqrt{4x(1-x)}$ to 1/$\sqrt{x-x^2}$, cso **A1**

(c) Answer: d/dx[arccos(1-2x)] = 1/$\sqrt{x-x^2}$ (printed result)

Question 2

(a) differentiates sinh(y) = x implicitly to get cosh(y) dy/dx = 1 **M1**

(a) uses $\cosh^2(y) - \sinh^2(y) = 1$ to write cosh(y) = $\sqrt{1+\sinh^2(y)}$ **M1**

(a) substitutes sinh(y) = x to get cosh(y) = $\sqrt{1+x^2}$ **M1**

(a) dy/dx = 1/$\sqrt{1+x^2}$, cso (printed result) **A1**

(a) Answer: dy/dx = 1/$\sqrt{1+x^2}$ (printed result)

(b) applies the chain rule with u = 2x, du/dx = 2 **M1**

(b) dy/dx = 2/$\sqrt{1+4x^2}$, oe **A1**

(b) Answer: dy/dx = 2/$\sqrt{1+4x^2}$

(c) sets sinh(y) = 2x, writes (e^y - e^{-y})/2 = 2x and forms e^{2y} - 4x e^y - 1 = 0 **M1**

(c) solves the quadratic in e^y, taking the positive root e^y = 2x + $\sqrt{4x^2+1}$ **M1**

(c) arsinh(2x) = ln(2x + $\sqrt{4x^2+1}$), cso (printed result) **A1**

(c) substitutes x=2 to get arsinh(4) = ln(4+$\sqrt{17}$), awrt 2.09 **A1**

(c) Answer: arsinh(2x) = ln(2x+$\sqrt{4x^2+1}$) (printed result); arsinh(4) = ln(4+$\sqrt{17}$) = awrt 2.09

Question 3

(a) recognises the standard form 1/(x^2 + a^2) with a = 1 **M1**

(a) integrates: (1) arctan(x) **M1**

(a) (1) arctan($\sqrt{3}$) = (1)(π/3) **A1**

(a) π/3 **A1**

(a) Answer: π/3

Question 4

(a) chooses u=x^n, dv/dx = e^x, so du/dx = n*x^{n-1}, v = e^x **M1**

(a) applies integration by parts: I_n = [x^n e^x]_0^1 - n*integral of x^{n-1} e^x dx **M1**

(a) evaluates boundary term [x^n e^x]_0^1 = e - 0 = e **A1**

(a) I_n = e - n*I_{n-1}, cso (printed result) **A1**

(a) Answer: I_n = e - n*I_{n-1} (printed result)

(b) states I_0 = integral from 0 to 1 of e^x dx = [e^x]_0^1 **M1**

(b) I_0 = e - 1, cao **A1**

(b) Answer: I_0 = e - 1

(c) uses I_1 = e - 1*I_0 = e-(e-1) = 1 **M1**

(c) uses I_2 = e - 2*I_1 = e-2 **M1**

(c) uses I_3 = e - 3*I_2 = e-3(e-2) = 6-2e **A1**

(c) I_3 = 6 - 2e, cao (awrt 0.563) **A1**

(c) Answer: I_3 = 6 - 2e (awrt 0.563)

Question 5

(a) integrates to get [-1/x]_1^R = -1/R - (-1) = 1-1/R (printed result) **M1**

(a) considers the limit as R tends to infinity, noting 1/R tends to 0 **M1**

(a) concludes the integral converges to 1, cso **A1**

(a) Answer: Integral converges to 1

(b) integrates to get integral from 1 to R of $x^{-1/2}$ dx = [2*$\sqrt{x}$]_1^R **M1**

(b) evaluates to 2*$\sqrt{R}$ - 2 **M1**

(b) considers limit as R tends to infinity and notes 2*$\sqrt{R}$ tends to infinity **A1**

(b) correct conclusion that the integral diverges, with justification **A1**

(b) Answer: Integral diverges (2*$\sqrt{R}$ -2 tends to infinity as R tends to infinity)

(c) considers integral from x=epsilon to x=1 of $x^{-1/2}$ dx = [2*$\sqrt{x}$]_epsilon^1 **M1**

(c) evaluates to 2 - 2*$\sqrt{epsilon}$ **M1**

(c) takes the limit as epsilon tends to 0+ and notes 2*$\sqrt{epsilon}$ tends to 0 **A1**

(c) concludes the integral converges to 2, cso **A1**

(c) Answer: Integral converges to 2

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Question 6

(a) differentiates to get $dy/dx = \sinh(x/5)$ **M1**

(a) uses $\cosh^2(x/5) - \sinh^2(x/5) = 1$ to show $1+\sinh^2(x/5) = \cosh^2(x/5)$, cso (printed result) **A1**

(a) Answer: $dy/dx = \sinh(x/5)$; $1+(dy/dx)^2 = \cosh^2(x/5)$ (printed result)

(b) states arc length = integral from a to b of $\sqrt{1+(dy/dx)^2}$ dx **B1**

(b) uses $\cosh(x/5) > 0$ to take the positive square root, $\sqrt{\cosh^2(x/5)} = \cosh(x/5)$ **B1**

(b) Answer: Arc length = integral from -4 to 4 of cosh(x/5) dx

(c) integrates to get $[5*\sinh(x/5)]_{-4}$^4 **M1**

(c) uses sinh being an odd function to write $5[\sinh(4/5)-\sinh(-4/5)] = 10*\sinh(0.8)$ **M1**

(c) exact form $10*\sinh(0.8)$ (or equivalent using exponentials), cao **A1**

(c) evaluates to awrt 8.88 metres **A1**

(c) Answer: Length = 10*sinh(0.8) metres, awrt 8.88 m

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