1.4 Further Calculus

EDEXCEL 9FM0 · Calculator allowed · about 210 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The region R is bounded by the curve $y = \sin(2x)$, for $0 \leq x \leq \pi/2$, and the x axis. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 1 is 4 marks)

2 The region R is bounded by the curve $y = e^{4x}$, the coordinate axes and the line $x = 1$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(3)**

(Total for Question 2 is 3 marks)

3 This question is about improper integrals.

(a) Find the value of the integral from 0 to infinity of $2e^{-2x}$ dx. **(3)**

(Total for Question 3 is 3 marks)

4 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = e^{4x}$ over the interval $2 \leq x \leq 5$, giving your answer in an exact form. **(3)**

(Total for Question 4 is 3 marks)

5 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to $4\sqrt{3}/2$ of $1/\sqrt{16 - x^2}$ dx. **(4)**

(Total for Question 5 is 4 marks)

6 The region R is bounded by the curve $y = x^2 + 2$, the coordinate axes and the line $x = 2$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 6 is 4 marks)

7 The region R is bounded by the curve $y = \sqrt{5x}$, the x axis and the line $x = 4$. The region R is rotated through 2pi radians about the x axis.

(a) Write down an integral for the volume of the solid formed, and evaluate it exactly. **(4)**

(Total for Question 7 is 4 marks)

8 The region R is bounded by the curve $y = x^2 + 3$, the coordinate axes and the line $x = 4$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 8 is 4 marks)

9 This question is about improper integrals.

(a) Use integration by parts to find the exact value of the integral from 0 to infinity of $3x\, e^{-4x}\, dx$. **(4)**

(Total for Question 9 is 4 marks)

10 This question is about improper integrals.

(a) Find the value of the integral from 0 to infinity of $3e^{-4x}$ dx. **(3)**

(Total for Question 10 is 3 marks)

11 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = x^2 + 3x$ over the interval $2 \leq x \leq 3$. **(4)**

(Total for Question 11 is 4 marks)

12 This question is about improper integrals.

(a) Evaluate the improper integral of $1/\sqrt{x}$ from $x = 0$ to $x = 1$. **(3)**

(Total for Question 12 is 3 marks)

13 The region R is bounded by the curve $y = \sqrt{6x}$, the x axis and the line $x = 4$. The region R is rotated through 2pi radians about the x axis.

(a) Write down an integral for the volume of the solid formed, and evaluate it exactly. **(4)**

(Total for Question 13 is 4 marks)

14 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = \sin(4x)$ over the interval $0 \leq x \leq \pi/4$. **(3)**

(Total for Question 14 is 3 marks)

15 The region R is bounded by the curve $y = 2\sin(3x)$, for $0 \leq x \leq \pi/3$, and the x axis. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(4)**

(Total for Question 15 is 4 marks)

16 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = \sin(2x)$ over the interval $0 \leq x \leq \pi/2$. **(3)**

(Total for Question 16 is 3 marks)

17 This question is about improper integrals.

(a) Evaluate the improper integral of $5/\sqrt{x}$ from $x = 0$ to $x = 1$. **(3)**

(Total for Question 17 is 3 marks)

18 This question concerns standard integrals leading to inverse trigonometric and inverse hyperbolic functions.

(a) Find the exact value of the integral: integral of $1/\sqrt{9-x^2}$ dx. **(3)**

(b) Find the integral: integral of $1/\sqrt{x^2+16}$ dx. **(3)**

(c) Hence evaluate integral from x=0 to x=4 of $1/\sqrt{25-x^2}$ dx, giving your answer to 3 significant figures. **(4)**

(Total for Question 18 is 10 marks)

19 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = 2x^2 + 3x$ over the interval $2 \leq x \leq 5$. **(4)**

(Total for Question 19 is 4 marks)

20 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = e^{4x}$ over the interval $0 \leq x \leq 4$, giving your answer in an exact form. **(3)**

(Total for Question 20 is 3 marks)

21 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to $2\sqrt{3}$ of $1/(x^2 + 36)$ dx. **(4)**

(Total for Question 21 is 4 marks)

22 This question is about improper integrals.

(a) Show that the improper integral of $1/x$ from $x = 1$ to infinity does not converge. **(3)**

(Total for Question 22 is 3 marks)

23 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to 2 of $1/(x^2 + 4)$ dx. **(4)**

(Total for Question 23 is 4 marks)

24 The region R is bounded by the curve $y = e^x$, the coordinate axes and the line $x = 3$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(3)**

(Total for Question 24 is 3 marks)

25 This question is about integrals that lead to inverse trigonometric functions.

(a) Find the exact value of the integral from 0 to $\sqrt{3}/2$ of $1/\sqrt{1 - x^2}$ dx. **(4)**

(Total for Question 25 is 4 marks)

26 The region R is bounded by the curve $y = e^{2x}$, the coordinate axes and the line $x = 3$. R is rotated through 2pi radians about the x axis.

(a) Find the exact volume of the solid generated. **(3)**

(Total for Question 26 is 3 marks)

27 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = 2/x$ over the interval $2 \leq x \leq 4$, in exact form. **(3)**

(Total for Question 27 is 3 marks)

28 This question is about improper integrals.

(a) Show that the improper integral of $5/x^4$ from $x = 1$ to infinity converges, and find its value. **(4)**

(Total for Question 28 is 4 marks)

29 This question concerns integration of a quadratic under a square root by completing the square.

(a) Show that $4x^2 + 8x + 13 = 4(x+1)^2 + 9$. **(2)**

(b) Hence find integral of $1/\sqrt{4x^2+8x+13}$ dx, giving your answer in the form k*arsinh(f(x)) + c. **(4)**

(c) Hence evaluate integral from x=0 to x=1 of $1/\sqrt{4x^2+8x+13}$ dx, giving your answer to 3 significant figures. **(4)**

(Total for Question 29 is 10 marks)

30 This question is about the mean value of a function.

(a) Find the mean value of $f(x) = e^x$ over the interval $0 \leq x \leq 4$, giving your answer in an exact form. **(3)**

(Total for Question 30 is 3 marks)

31 Let $f(x) = 1/\sqrt{4-x^2}$ for $0 \leq x \leq \sqrt{2}$.

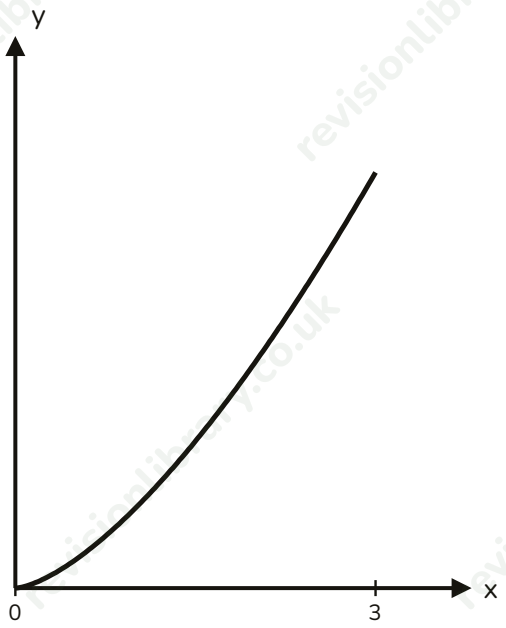
(a) State the formula for the mean value of a function f(x) over the interval [a, b]. **(1)**

(b) Find the mean value of f(x) over the interval $[0, \sqrt{2}]$, giving your answer as an exact expression involving π and a surd. **(4)**

(c) Determine, showing your working, whether f(1) exceeds the mean value found in part (b). **(3)**

(Total for Question 31 is 8 marks)

32 A curve C has equation $y = (2/3)\, x^{3/2}$, for $0 \leq x \leq 3$.



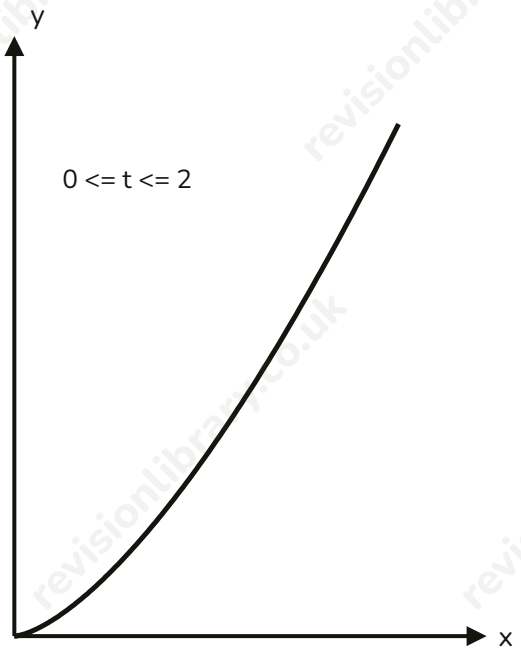
(a) Find dy/dx, and show that $1 + (dy/dx)^2 = 1 + x$. **(3)**

(b) State the formula used to find the arc length of a curve $y=f(x)$ between $x=a$ and $x=b$. **(1)**

(c) Hence find the exact length of the arc of C between $x=0$ and $x=3$, giving your answer as a fraction in its simplest form. **(4)**

(Total for Question 32 is 8 marks)

33 A curve is defined parametrically by $x = 3t^2$, $y = 2t^3$, for $0 \leq t \leq 2$.



(a) Find dx/dt and dy/dt, and show that $(dx/dt)^2 + (dy/dt)^2 = 36t^2(1+t^2)$. **(3)**

(b) State the formula for the arc length of a curve defined parametrically for $\alpha \leq t \leq \beta$, and use it, together with the fact that $t \geq 0$ on this curve, to show that the arc length is given by integral from t=0 to t=2 of $6t*\sqrt{1+t^2}$ dt. **(2)**

(c) Hence show that the exact arc length of the curve for $0 \leq t \leq 2$ is $10*\sqrt{5} - 2$, and find this length to 3 significant figures. **(5)**

(Total for Question 33 is 10 marks)

34 The curve $y = \sqrt{x}$, for $1 \leq x \leq 4$, is rotated through $2*\pi$ radians about the x-axis to form a surface of revolution.

(a) Find dy/dx, and show that $1 + (dy/dx)^2 = (4x+1)/(4x)$. **(3)**

(b) Using the formula $S = 2*\pi*$integral of $y*\sqrt{1+(dy/dx)^2}$ dx for a surface of revolution about the x-axis, show that $S = \pi*$integral from x=1 to x=4 of $\sqrt{4x+1}$ dx. **(2)**

(c) Hence find the exact surface area generated, giving your answer in the form $(\pi/6)(a*\sqrt{a} - b*\sqrt{b})$, and evaluate it to 3 significant figures. **(5)**

(Total for Question 34 is 10 marks)

35 Let I_n = integral from x=0 to x=π/2 of $\sin^n(x)\,dx$, for $n \geq 0$.

(a) Using integration by parts on $\sin^n(x) = \sin(x)*\sin^{n-1}(x)$, show that, for $n \geq 2$, $I_n = ((n-1)/n) * I_{n-2}$. **(5)**

(b) State the exact values of I_0 and I_1. **(2)**

(c) Hence find the exact value of I_5, giving your answer as a fraction in its simplest form, and state its value correct to 3 significant figures. **(4)**

(Total for Question 35 is 11 marks)

Mark scheme · 1.4 Further Calculus

Question 1

(a) $V = \pi$ integral of y^2 dx **M1**

(a) uses $\sin^2(2x) = (1 - \cos(4x))/2$ **M1**

(a) integrates and substitutes the limits 0 and $\pi/2$ **M1**

(a) $V = \pi^2/4$ **A1**

(a) Answer: $\pi^2/4$

Question 2

(a) $V = \pi$ integral of e^{8x} dx **M1**

(a) integrates: $\pi[e^{8x}/8]$ **M1**

(a) substitutes the limits: $(\pi/8)(e^8 - 1)$ **A1**

(a) Answer: $(\pi/8)(e^8 - 1)$

Question 3

(a) integrates: $-(2/2)e^{-2x}$ **M1**

(a) takes the limit as the upper bound tends to infinity **M1**

(a) 1 **A1**

(a) Answer: 1

Question 4

(a) mean value = (1/(b - a)) integral of f **M1**

(a) integrates to $e^{4x}/4$ **M1**

(a) $(e^{20} - e^8)/12$ **A1**

(a) Answer: $(e^{20} - e^8)/12$

Question 5

(a) recognises the standard form $1/\sqrt{a^2 - x^2}$ with a = 4 **M1**

(a) integrates: arcsin(x/4) **M1**

(a) $\arcsin(\sqrt{3}/2) - \arcsin(0)$ **A1**

(a) $\pi/3$ **A1**

(a) Answer: $\pi/3$

Question 6

(a) $V = \pi$ integral of $(x^2 + 2)^2$ dx **M1**

(a) expands: $x^4 + 4x^2 + 4$ **M1**

(a) integrates term by term and substitutes the limits **M1**

(a) V = 376pi/15 **A1**

(a) Answer: 376pi/15

Question 7

(a) $V = \pi$ integral of y^2 dx **M1**

(a) π integral from 0 to 4 of 5x dx **A1**

(a) integrates: $\pi[5/2x^2]$ **M1**

(a) V = 40pi **A1**

(a) Answer: 40pi

Question 8

(a) $V = \pi$ integral of $(x^2 + 3)^2$ dx **M1**

(a) expands: $x^4 + 6x^2 + 9$ **M1**

(a) integrates term by term and substitutes the limits **M1**

(a) V = 1844pi/5 **A1**

(a) Answer: 1844pi/5

Question 9

(a) u = x, $dv/dx = e^{-4x}$ **M1**

(a) $-x\,e^{-4x}/4 +$ integral of $e^{-4x}/4$ dx **A1**

(a) uses $x\,e^{-kx}$ tends to 0 as x tends to infinity **M1**

(a) 3/16 **A1**

(a) Answer: 3/16

Question 10

(a) integrates: $-(3/4)e^{-4x}$ **M1**

(a) takes the limit as the upper bound tends to infinity **M1**

(a) 3/4 **A1**

(a) Answer: 3/4

Question 11

(a) mean value = (1/(3 - 2)) integral of f(x) dx **M1**

(a) integrates: $1/3x^3 + 3/2x^2$ **M1**

(a) integral = 83/6 **A1**

(a) mean value = 83/6 **A1**

(a) Answer: 83/6

Question 12

(a) integrates: $2x^{1/2}$ **M1**

(a) replaces the lower limit by t and lets t tend to 0 from above **M1**

(a) 2 **A1**

(a) Answer: 2

Question 13

(a) $V = \pi$ integral of y^2 dx **M1**

(a) π integral from 0 to 4 of 6x dx **A1**

(a) integrates: $\pi[3x^2]$ **M1**

(a) V = 48pi **A1**

(a) Answer: 48pi

Question 14

(a) mean value = (1/(π/4)) integral of sin(4x) dx **M1**

(a) integral = [-cos(4x)/4] from 0 to $\pi/4$ = 2/4 **M1**

(a) mean value = $2/\pi$ **A1**

(a) Answer: $2/\pi$

Question 15

(a) $V = \pi$ integral of y^2 dx **M1**

(a) uses $\sin^2(3x) = (1 - \cos(6x))/2$ **M1**

(a) integrates and substitutes the limits 0 and $\pi/3$ **M1**

(a) $V = 2pi^2/3$ **A1**

(a) Answer: $2pi^2/3$

Question 16

(a) mean value = (1/(π/2)) integral of sin(2x) dx **M1**

(a) integral = [-cos(2x)/2] from 0 to $\pi/2$ = 2/2 **M1**

(a) mean value = $2/\pi$ **A1**

(a) Answer: $2/\pi$

Question 17

(a) integrates: $10x^{1/2}$ **M1**

(a) replaces the lower limit by t and lets t tend to 0 from above **M1**

(a) 10 **A1**

(a) Answer: 10

Question 18

(a) recognises the standard form integral $1/\sqrt{a^2-x^2}$ dx = arcsin(x/a) + c with a=3 **M1**

(a) arcsin(x/3) seen **A1**

(a) + c included, cao **A1**

(a) Answer: arcsin(x/3) + c

(b) recognises the standard form integral $1/\sqrt{x^2+a^2}$ dx = arsinh(x/a) + c with a=4 **M1**

(b) arsinh(x/4) seen **A1**

(b) + c included, oe (accept equivalent log form $\ln(x+\sqrt{x^2+16})+k$) **A1**

(b) Answer: arsinh(x/4) + c

(c) writes the integral as [arcsin(x/5)] with limits 0 to 4 **M1**

(c) substitutes limits to get arcsin(4/5) - arcsin(0) **M1**

(c) exact value arcsin(4/5) (or arcsin(0.8)), oe **A1**

(c) awrt 0.927 (radians) **A1**

(c) Answer: arcsin(4/5) = awrt 0.927

Question 19

(a) mean value = (1/(5 - 2)) integral of f(x) dx **M1**

(a) integrates: $2/3x^3 + 3/2x^2$ **M1**

(a) integral = 219/2 **A1**

(a) mean value = 73/2 **A1**

(a) Answer: 73/2

Question 20

(a) mean value = (1/(b - a)) integral of f **M1**

(a) integrates to $e^{4x}/4$ **M1**

(a) $(e^{16} - 1)/16$ **A1**

(a) Answer: $(e^{16} - 1)/16$

Question 21

(a) recognises the standard form $1/(x^2 + a^2)$ with a = 6 **M1**

(a) integrates: (1/6) arctan(x/6) **M1**

(a) (1/6) arctan($1/\sqrt{3}$) = (1/6)(π/6) **A1**

(a) π/36 **A1**

(a) Answer: π/36

Question 22

(a) integrates: 1 ln x **M1**

(a) evaluates between 1 and t **M1**

(a) 1 ln t tends to infinity as t tends to infinity, so the integral diverges **A1**

(a) Answer: diverges

Question 23

(a) recognises the standard form $1/(x^2 + a^2)$ with a = 2 **M1**

(a) integrates: (1/2) arctan(x/2) **M1**

(a) (1/2) arctan(1) = (1/2)(π/4) **A1**

(a) π/8 **A1**

(a) Answer: π/8

Question 24

(a) V = π integral of e^{2x} dx **M1**

(a) integrates: $\pi[e^{2x}/2]$ **M1**

(a) substitutes the limits: $(\pi/2)(e^6 - 1)$ **A1**

(a) Answer: $(\pi/2)(e^6 - 1)$

Question 25

(a) recognises the standard form $1/\sqrt{a^2 - x^2}$ with a = 1 **M1**

(a) integrates: arcsin(x) **M1**

(a) arcsin($\sqrt{3}/2$) - arcsin(0) **A1**

(a) π/3 **A1**

(a) Answer: π/3

Question 26

(a) V = π integral of e^{4x} dx **M1**

(a) integrates: $\pi[e^{4x}/4]$ **M1**

(a) substitutes the limits: $(\pi/4)(e^{12} - 1)$ **A1**

(a) Answer: $(\pi/4)(e^{12} - 1)$

Question 27

(a) mean value = (1/(b - a)) integral of f **M1**

(a) integral = 2 ln(2) **A1**

(a) mean value = ln(2) **A1**

(a) Answer: ln(2)

Question 28

(a) integrates: $5x^{-3}/(-3) = -5/3x^{-3}$ **M1**

(a) replaces the upper limit by t and lets t tend to infinity **M1**

(a) t^{-3} tends to 0 so the integral converges **A1**

(a) value = 5/3 **A1**

(a) Answer: 5/3

Question 29

(a) writes $4x^2+8x+13 = 4(x^2+2x)+13 = 4[(x+1)^2 - 1] + 13$ **M1**

(a) simplifies to $4(x+1)^2 + 9$, cso (printed result) **A1**

(a) Answer: $4x^2+8x+13 = 4(x+1)^2+9$ (printed result)

(b) writes $\sqrt{4x^2+8x+13} = \sqrt{4(x+1)^2+9} = 2\sqrt{(x+1)^2+9/4}$ **M1**

(b) uses standard form $1/\sqrt{u^2+a^2}$ du = arsinh(u/a)+c with u=x+1, a=3/2 **M1**

(b) (1/2) arsinh(2(x+1)/3) seen, oe **A1**

(b) + c included, cao **A1**

(b) Answer: (1/2) arsinh(2(x+1)/3) + c

(c) substitutes limits into (1/2)arsinh(2(x+1)/3), giving (1/2) [arsinh(4/3) - arsinh(2/3)] **M1**

(c) uses arsinh(4/3) = ln(3) (since 4/3+5/3=3) or equivalent numeric evaluation **M1**

(c) arsinh(2/3) evaluated correctly, awrt 0.625 **A1**

(c) final answer awrt 0.237 **A1**

(c) Answer: awrt 0.237

Question 30

(a) mean value = (1/(b - a)) integral of f **M1**

(a) integrates to e^x **M1**

(a) $(e^4 - 1)/4$ **A1**

(a) Answer: $(e^4 - 1)/4$

Question 31

(a) mean value = (1/(b-a)) * integral from a to b of f(x) dx, oe **B1**

(a) Answer: Mean value = (1/(b-a)) integral from a to b of f(x) dx

(b) writes mean value = $(1/(\sqrt{2}-0))$ * integral from 0 to $\sqrt{2}$ of $1/\sqrt{4-x^2}$ dx **M1**

(b) integrates to arcsin(x/2), and substitutes limits to get $\arcsin(\sqrt{2}/2) - \arcsin(0)$ **M1**

(b) recognises $\arcsin(\sqrt{2}/2) = \pi/4$, giving integral = $\pi/4$ **A1**

(b) mean value = $(\pi*\sqrt{2})/8$, oe cao **A1**

(b) Answer: Mean value = $(\pi*\sqrt{2})/8$ (awrt 0.555)

(c) evaluates f(1) = $1/\sqrt{4-1} = 1/\sqrt{3}$, awrt 0.577 **M1**

(c) compares with mean value awrt 0.555 from part (b) **A1**

(c) correct conclusion that f(1) exceeds the mean value, ft from parts (b) **A1**

(c) Answer: f(1) = $1/\sqrt{3}$ = awrt 0.577, which exceeds the mean value of awrt 0.555, so f(1) exceeds the mean value.

Question 32

(a) differentiates to get dy/dx = $(2/3)*(3/2)*x^{1/2}$ **M1**

(a) dy/dx = $\sqrt{x}$, oe **A1**

(a) $1 + (dy/dx)^2 = 1 + (\sqrt{x})^2 = 1+x$, cso (printed result) **A1**

(a) Answer: dy/dx = $\sqrt{x}$; $1+(dy/dx)^2 = 1+x$ (printed result)

(b) arc length = integral from a to b of $\sqrt{1+(dy/dx)^2}$ dx, oe **B1**

(b) Answer: Arc length = integral from a to b of $\sqrt{1+(dy/dx)^2}$ dx

(c) sets up arc length = integral from 0 to 3 of $\sqrt{1+x}$ dx **M1**

(c) integrates to $(2/3)(1+x)^{3/2}$ **M1**

(c) substitutes limits to get $(2/3)(4^{3/2} - 1^{3/2}) = (2/3)(8-1)$ **A1**

(c) arc length = 14/3, cao **A1**

(c) Answer: Arc length = 14/3 (awrt 4.67)

Question 33

(a) differentiates to get dx/dt = 6t, $dy/dt = 6t^2$ **M1**

(a) squares and adds: $36t^2 + 36t^4$ **M1**

(a) factorises to $36t^2(1+t^2)$, cso (printed result) **A1**

(a) Answer: dx/dt = 6t, $dy/dt = 6t^2$; $(dx/dt)^2+(dy/dt)^2 = 36t^2(1+t^2)$ (printed result)

(b) states arc length = integral from α to β of $\sqrt{(dx/dt)^2+(dy/dt)^2}$ dt **B1**

(b) uses t≥0 to take the positive square root, $\sqrt{36t^2(1+t^2)} = 6t*\sqrt{1+t^2}$ **B1**

(b) Answer: Arc length = integral from 0 to 2 of $6t*\sqrt{1+t^2}$ dt

(c) uses substitution $u=1+t^2$, du=2t dt to rewrite the integral as 3*integral of $\sqrt{u}$ du **M1**

(c) integrates to get $2u^{3/2}$ **M1**

(c) changes limits (t=0 to u=1, t=2 to u=5) and substitutes to get $2(5^{3/2} - 1)$ **A1**

(c) simplifies to $10*\sqrt{5} - 2$, cso (printed result) **A1**

(c) evaluates to awrt 20.4 **A1**

(c) Answer: Arc length = $10*\sqrt{5} - 2$ (printed result), awrt 20.4

Question 34

(a) differentiates to get dy/dx = $1/(2*\sqrt{x})$ **M1**

(a) squares to get $(dy/dx)^2 = 1/(4x)$ **M1**

(a) $1+1/(4x) = (4x+1)/(4x)$, cso (printed result) **A1**

(a) Answer: dy/dx = $1/(2\sqrt{x})$; $1+(dy/dx)^2 = (4x+1)/(4x)$ (printed result)

(b) substitutes $y=\sqrt{x}$ and the result from part (a) into the surface area formula **M1**

(b) simplifies $\sqrt{x}*\sqrt{(4x+1)/(4x)} = \sqrt{4x+1}/2$ to reach S = π*integral of $\sqrt{4x+1}$ dx, cso (printed result) **A1**

(b) Answer: S = π*integral from 1 to 4 of $\sqrt{4x+1}$ dx (printed result)

(c) integrates $\sqrt{4x+1}$ to get $(1/6)(4x+1)^{3/2}$ **M1**

(c) substitutes upper limit x=4: $(1/6)(17)^{3/2} = (1/6)(17*\sqrt{17})$ **M1**

(c) substitutes lower limit x=1: $(1/6)(5)^{3/2} = (1/6)(5*\sqrt{5})$ **A1**

(c) S = $(\pi/6)(17*\sqrt{17} - 5*\sqrt{5})$, cso **A1**

(c) evaluates to awrt 30.8 **A1**

(c) Answer: S = $(\pi/6)(17*\sqrt{17} - 5*\sqrt{5})$ (awrt 30.8)

Question 35

(a) chooses $u=\sin^{n-1}(x)$, dv/dx=sin(x), so $du/dx=(n-1)\sin^{n-2}(x)\cos(x)$, $v=-\cos(x)$ **M1**

(a) applies parts and evaluates boundary term $[-\cos(x)\sin^{n-1}(x)]_0^{\pi/2} = 0$ **M1**

(a) uses $\cos^2(x) = 1-\sin^2(x)$ to write (n-1)*integral of $(1-\sin^2(x))\sin^{n-2}(x)$ dx **M1**

(a) expands to $I_n = (n-1)(I_{n-2} - I_n)$ and rearranges to collect I_n terms: $n*I_n = (n-1)*I_{n-2}$ **M1**

(a) $I_n = ((n-1)/n)*I_{n-2}$, cso (printed result) **A1**

(a) Answer: $I_n = ((n-1)/n) * I_{n-2}$ (printed result)

(b) $I_0 = \pi/2$ **B1**

(b) $I_1 = 1$ **B1**

(b) Answer: $I_0 = \pi/2$, $I_1 = 1$

(c) uses $I_3 = (2/3)*I_1 = 2/3$ **M1**

(c) uses $I_5 = (4/5)*I_3$ **M1**

(c) $I_5 = 8/15$, cao **A1**

(c) awrt 0.533 **A1**

(c) Answer: $I_5 = 8/15$ (awrt 0.533)