1.5 Further Vectors: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A triangular solar panel has corners at the points P(1, 0, 2), Q(3, 4, 1) and R(0, 2, 5), with coordinates measured in metres.

(a) Find the vectors PQ and PR. **(2)**

(b) Calculate the vector product PQ x PR. **(3)**

(c) Hence find the area of triangle PQR, giving your answer in m^2 to 3 significant figures. **(3)**

(Total for Question 1 is 8 marks)

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2 A line l has equation r = (1, 0, 2) + t(2, 1, -2). A navigation beacon is located at E(5, 4, -1).

(a) Find the position vector of the foot of the perpendicular from E to l. **(5)**

(b) Hence find the position vector of the reflection of E in the line l. **(2)**

(Total for Question 2 is 7 marks)

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3 The lines l_1 and l_2 have vector equations r = (6i + 9j - 3k) + λ(-i - 2j) and r = (11j - 9k) + μ(2i - 3j + 3k) respectively.

(a) Show that l_1 and l_2 intersect, and find the position vector of their point of intersection. **(5)**

(b) Find the acute angle between the lines, to 1 decimal place. **(3)**

(Total for Question 3 is 8 marks)

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4 The line l has equation r = (-3k) + t(-i + 2j + 3k) and the point Q has position vector -i - j + 6k.

(a) Find the value of t for which the point on l is closest to Q. **(3)**

(b) Hence find the position vector of the foot of the perpendicular from Q to l, and the shortest distance from Q to l, to 3 significant figures. **(4)**

(Total for Question 4 is 7 marks)

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5 The line l has equation $\mathbf{r} = (4\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(-3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ and the point Q has position vector $-2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

(a) Find the value of t for which the point on l is closest to Q. **(3)**

(b) Hence find the position vector of the foot of the perpendicular from Q to l, and the shortest distance from Q to l, to 3 significant figures. **(4)**

(Total for Question 5 is 7 marks)

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6 The points A, B and C have position vectors $4\mathbf{j} - 2\mathbf{k}$, $-\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $-\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ respectively, relative to a fixed origin O.

(a) Find the vector products AB x AC, and hence a vector perpendicular to the plane ABC (simplified). **(4)**

(b) Find a Cartesian equation of the plane ABC in the form $ax + by + cz = d$. **(3)**

(c) Find the perpendicular distance from the point Q with position vector $-4\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$ to the plane ABC, to 3 significant figures. **(3)**

(d) The line with equation $\mathbf{r} = (-2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) + t(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ meets the plane ABC at the point R. Find the position vector of R. **(3)**

(Total for Question 6 is 13 marks)

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Mark scheme · 1.5 Further Vectors: 1-hour Pack

Question 1

(a) attempts Q-P and R-P **M1**

(a) PQ = (2,4,-1) and PR = (-1,2,3), both cao **A1**

(a) Answer: PQ = (2, 4, -1), PR = (-1, 2, 3)

(b) sets up the determinant/component method for the vector product **M1**

(b) at least two components correct **A1**

(b) PQ x PR = (14, -5, 8), fully correct, cao **A1**

(b) Answer: PQ x PR = (14, -5, 8)

(c) finds $|PQ \times PR| = \sqrt{14^2+(-5)^2+8^2}$ **M1**

(c) uses area = (1/2)|PQ x PR| **M1**

(c) awrt 8.44 m^2 **A1**

(c) Answer: 8.44 m^2 (3 sf)

Question 2

(a) writes a general point on l as (1+2t, t, 2-2t) **M1**

(a) forms the vector from this point to E **M1**

(a) sets this vector's dot product with (2,1,-2) equal to zero **M1**

(a) solves to get t = 2 **M1**

(a) foot of perpendicular = (5, 2, -2), cao **A1**

(a) Answer: (5, 2, -2)

(b) uses E' = 2F - E where F is the foot of the perpendicular found in (a) **M1**

(b) E' = (5, 0, -3), ft from (a) **A1**

(b) Answer: (5, 0, -3)

Question 3

(a) equates the i, j and k components to form simultaneous equations in λ and μ **M1**

(a) solves two of the equations **M1**

(a) λ = 2 and μ = 2 **A1**

(a) checks the third equation is satisfied, so the lines intersect **B1**

(a) position vector 4i + 5j - 3k **A1**

(a) Answer: 4i + 5j - 3k

(b) uses cos θ = |d1 . d2|/(|d1||d2|) **M1**

(b) d1 . d2 = 4 **A1**

(b) θ = 67.6 degrees **A1**

(b) Answer: 67.6 degrees

Question 4

(a) the vector from the point on the line to Q must be perpendicular to the direction vector of l **M1**

(a) sets (Q - A - t d) . d = 0 **M1**

(a) t = 26/14 = 13/7 **A1**

(a) Answer: t = 13/7

(b) substitutes t into the equation of l **M1**

(b) -(13/7)i + (26/7)j + (18/7)k **A1**

(b) distance = |Q - F| **M1**

(b) 5.89 **A1**

(b) Answer: -(13/7)i + (26/7)j + (18/7)k; 5.89

Question 5

(a) the vector from the point on the line to Q must be perpendicular to the direction vector of l **M1**

(a) sets (Q - A - t d) . d = 0 **M1**

(a) t = 11/22 = 1/2 **A1**

(a) Answer: t = 1/2

(b) substitutes t into the equation of l **M1**

(b) (5/2)i + (1/2)j + k **A1**

(b) distance = |Q - F| **M1**

(b) 5.96 **A1**

(b) Answer: (5/2)i + (1/2)j + k; 5.96

Question 6

(a) finds AB and AC **M1**

(a) AB = -i - j + k, AC = -i - j - 2k **A1**

(a) evaluates the vector product with the determinant method **M1**

(a) AB x AC = 3i - 3j, so a normal is i - j **A1**

(a) Answer: normal i - j

(b) uses n . r = n . a **M1**

(b) d = -4 **A1**

(b) x - y = -4 **A1**

(b) Answer: x - y = -4

(c) uses |n . q - d|/|n| **M1**

(c) $|-9 - (-4)|/\sqrt{2}$ **A1**

(c) 3.54 **A1**

(c) Answer: 3.54

(d) substitutes the parametric form of the line into the plane equation **M1**

(d) t = -5/4 **A1**

(d) -(23/4)i - (7/4)j + (1/2)k **A1**

(d) Answer: -(23/4)i - (7/4)j + (1/2)k

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