1.5 Further Vectors: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Freya is programming a drone's flight path, which she models as the line l with vector equation r = (4, -2, 1) + λ(3, -1, 2), where λ is a real parameter and distances are in metres.

(a) Write down the cartesian equations of l in the form (x-a)/p = (y-b)/q = (z-c)/r. **(2)**

(b) Find the coordinates of the point where l crosses the plane z = 7. **(3)**

(c) Find the coordinates of the point where l crosses the plane y = 0. **(2)**

(Total for Question 1 is 7 marks)

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2 A triangular plot of land for a proposed footbridge has corners at A(1, 2, -1), B(3, 0, 2) and C(-1, 4, 0), with coordinates measured in metres.

(a) Find the vectors AB and AC. **(2)**

(b) Find a vector normal to the plane containing A, B and C. **(3)**

(c) Show that the cartesian equation of the plane ABC can be written as x + y = 3. **(3)**

(Total for Question 2 is 8 marks)

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3 Three edges of a tetrahedral bracket meet at a common vertex and are represented by the vectors a = (2, 1, -1), b = (1, 3, 2) and c = (-1, 2, 4), with lengths in centimetres.

(a) Calculate the scalar triple product a . (b x c). **(4)**

(b) Hence find the volume of the tetrahedron with edges a, b and c meeting at the vertex, giving your answer to 3 significant figures. **(2)**

(Total for Question 3 is 6 marks)

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4 The points A, B and C have position vectors 2i - j + k, j and 3i - k respectively, relative to a fixed origin O.

(a) Find the vector products AB x AC, and hence a vector perpendicular to the plane ABC (simplified). **(4)**

(b) Find a Cartesian equation of the plane ABC in the form ax + by + cz = d. **(3)**

(c) Find the perpendicular distance from the point Q with position vector -2i - 4j - 3k to the plane ABC, to 3 significant figures. **(3)**

(d) The line with equation r = (-4i + 5j + 3k) + t(i - 2j - 3k) meets the plane ABC at the point R. Find the position vector of R. **(3)**

(Total for Question 4 is 13 marks)

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5 The lines l_1 and l_2 have vector equations r = (-i - 2j + k) + λ(2i + 3j + 2k) and r = (i + 3j - 3k) + μ(2i + 4j - k) respectively.

(a) Show that l_1 and l_2 intersect, and find the position vector of their point of intersection. **(5)**

(b) Find the acute angle between the lines, to 1 decimal place. **(3)**

(Total for Question 5 is 8 marks)

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6 Two straight underground cable routes are modelled by the lines l1: r = (1, 2, 0) + s(1, -1, 2) and l2: r = (0, 3, 1) + t(2, 1, -1), where distances are in metres.

(a) Show that l1 and l2 are skew lines. **(5)**

(b) Find the shortest distance between l1 and l2, giving your answer to 3 significant figures. **(5)**

(Total for Question 6 is 10 marks)

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Mark scheme · 1.5 Further Vectors: Test Pack

Question 1

(a) correct point used, e.g. (4,-2,1) appearing as a, b, c **B1**

(a) (x-4)/3 = (y+2)/-1 = (z-1)/2, oe **B1**

(a) Answer: (x-4)/3 = (y+2)/(-1) = (z-1)/2

(b) sets 1 + 2*λ = 7 **M1**

(b) λ = 3 **A1**

(b) point (13, -5, 7), cao **A1**

(b) Answer: (13, -5, 7)

(c) sets -2 - λ = 0 and solves for λ = -2 **M1**

(c) point (-2, 0, -3), cao **A1**

(c) Answer: (-2, 0, -3)

Question 2

(a) attempts B-A and C-A **M1**

(a) AB = (2,-2,3) and AC = (-2,2,1), both cao **A1**

(a) Answer: AB = (2, -2, 3), AC = (-2, 2, 1)

(b) attempts AB x AC **M1**

(b) unsimplified normal (-8, -8, 0), oe **A1**

(b) simplified normal, e.g. (1, 1, 0), oe (any nonzero scalar multiple accepted) **A1**

(b) Answer: Normal direction (1, 1, 0) (or any scalar multiple, e.g. (-8,-8,0))

(c) uses normal (1,1,0) with point A(1,2,-1) in n.(r-a)=0 **M1**

(c) expands to (x-1)+(y-2)+0(z+1)=0 **M1**

(c) correctly simplifies to x+y=3 (printed result), cso **A1**

(c) Answer: x + y = 3 (printed result)

Question 3

(a) attempts b x c **M1**

(a) b x c = (8, -6, 5), cao **A1**

(a) forms the dot product of a with their vector b x c **M1**

(a) a.(bxc) = 5, cao **A1**

(a) Answer: a . (b x c) = 5

(b) uses volume = (1/6)|a.(bxc)| **M1**

(b) awrt 0.833 cm^3 **A1**

(b) Answer: 5/6 = 0.833 cm^3 (3 sf)

Question 4

(a) finds AB and AC **M1**

(a) AB = -2i + 2j - k, AC = i + j - 2k **A1**

(a) evaluates the vector product with the determinant method **M1**

(a) AB x AC = -3i - 5j - 4k, so a normal is -3i - 5j - 4k **A1**

(a) Answer: normal -3i - 5j - 4k

(b) uses n . r = n . a **M1**

(b) d = -5 **A1**

(b) -3x - 5y - 4z = -5 **A1**

(b) Answer: -3x - 5y - 4z = -5

(c) uses |n . q - d|/|n| **M1**

(c) $|38 - (-5)|/(5\sqrt{2})$ **A1**

(c) 6.08 **A1**

(c) Answer: 6.08

(d) substitutes the parametric form of the line into the plane equation **M1**

(d) t = 20/19 **A1**

(d) -(56/19)i + (55/19)j - (3/19)k **A1**

(d) Answer: -(56/19)i + (55/19)j - (3/19)k

Question 5

(a) equates the i, j and k components to form simultaneous equations in λ and μ **M1**

(a) solves two of the equations **M1**

(a) λ = -1 and μ = -2 **A1**

(a) checks the third equation is satisfied, so the lines intersect **B1**

(a) position vector -3i - 5j - k **A1**

(a) Answer: -3i - 5j - k

(b) uses cos θ = |d1 . d2|/(|d1||d2|) **M1**

(b) d1 . d2 = 14 **A1**

(b) θ = 42.2 degrees **A1**

(b) Answer: 42.2 degrees

Question 6

(a) notes (1,-1,2) is not a scalar multiple of (2,1,-1) (e.g. ratios 1/2, -1/1, 2/-1 are not all equal), so the lines are not parallel **B1**

(a) equates components to form simultaneous equations, e.g. 1+s=2t and 2-s=3+t **M1**

(a) solves these two equations to get s=-1 and t=0 **M1**

(a) substitutes into the third equation, 2s=1-t, to get -2 = 1, a contradiction, cso **A1**

(a) concludes l1 and l2 do not intersect and are not parallel, so they are skew **B1**

(a) Answer: l1 and l2 are skew (not parallel, and shown not to intersect)

(b) finds the vector between points on the lines, (0,3,1)-(1,2,0) = (-1,1,1) **M1**

(b) attempts d1 x d2 **M1**

(b) d1 x d2 = (-1, 5, 3), cao **A1**

(b) applies distance = |(b-a).(d1xd2)| / |d1xd2| **M1**

(b) awrt 1.52 m **A1**

(b) Answer: 1.52 m (3 sf)

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