1.6 Polar Coordinates: 1-hour Pack

Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has polar equation $r = 6(1 + \cos\theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 120$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 1 is 7 marks)

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2 The curve C has polar equation $r = 1(2 + \cos\theta)$, for $0 \leq \theta < 2pi$.

(a) Find the exact area enclosed by C. **(4)**

(Total for Question 2 is 4 marks)

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3 The curve C has polar equation $r = 7(1 + \cos\theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 60$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 3 is 7 marks)

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4 The curve C has polar equation $r = 3\sin(2\theta)$, for $0 \leq \theta \leq \pi/2$. It forms one loop (petal) of a four-petal curve.

(a) Find the exact area enclosed by this loop. **(4)**

(Total for Question 4 is 4 marks)

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5 The curve C has polar equation $r = 7(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 120$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 5 is 7 marks)

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6 The curve C has polar equation $r = 2(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$. The region R is bounded by C, the initial line and the half line $\theta = \pi/2$.

(a) Find the exact area of R. **(4)**

(Total for Question 6 is 4 marks)

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7 The curve C has polar equation $r = 3(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 90$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 7 is 7 marks)

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8 A lemniscate has polar equation $r^2 = 4\cos(2\text{theta})$.

(a) State the two ranges of values of θ, for $0 \leq \theta < 2\text{pi}$, for which the curve is defined (i.e. $r^2 \geq 0$), describing the right-hand and left-hand loops. **(2)**

(b) Find the exact area enclosed by one loop of the curve. **(5)**

(c) Find the values of θ at which the curve has a tangent at the pole for the right-hand loop, and briefly interpret this result. **(3)**

(Total for Question 8 is 10 marks)

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Mark scheme · 1.6 Polar Coordinates: 1-hour Pack

Question 1

(a) r = 6(1 + cos 120 degrees) **M1**

(a) r = 3.00 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (-1.50, 2.60) **A1**

(a) Answer: r = 3.00, (-1.50, 2.60)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 12 at θ = 0 **A1**

(b) greatest distance from the pole = 12 **B1**

(b) Answer: Maximum r = 12 at θ = 0; greatest distance = 12.

Question 2

(a) area = (1/2) integral from 0 to 2pi of r^2 d θ **M1**

(a) expands 1(4 + 4cos θ + $\cos^2$ θ) **M1**

(a) integrates, using $\cos^2$ θ = (1 + cos 2theta)/2, and substitutes the limits **M1**

(a) 9pi/2 **A1**

(a) Answer: 9pi/2

Question 3

(a) r = 7(1 + cos 60 degrees) **M1**

(a) r = 10.5 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (5.25, 9.09) **A1**

(a) Answer: r = 10.5, (5.25, 9.09)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 14 at θ = 0 **A1**

(b) greatest distance from the pole = 14 **B1**

(b) Answer: Maximum r = 14 at θ = 0; greatest distance = 14.

Question 4

(a) area = (1/2) integral of r^2 d θ **M1**

(a) uses $\sin^2$(2theta) = (1 - cos 4theta)/2 **M1**

(a) integrates and substitutes the limits 0 and π/2 **M1**

(a) 9pi/8 **A1**

(a) Answer: 9pi/8

Question 5

(a) r = 7(1 + cos 120 degrees) **M1**

(a) r = 3.50 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (-1.75, 3.03) **A1**

(a) Answer: r = 3.50, (-1.75, 3.03)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 14 at θ = 0 **A1**

(b) greatest distance from the pole = 14 **B1**

(b) Answer: Maximum r = 14 at θ = 0; greatest distance = 14.

Question 6

(a) area = (1/2) integral from 0 to π/2 of r^2 d θ **M1**

(a) uses $\cos^2$ θ = (1 + cos 2theta)/2 and integrates **M1**

(a) substitutes the limits 0 and π/2 **M1**

(a) 4(3pi/8 + 1) **A1**

(a) Answer: 4(3pi/8 + 1)

Question 7

(a) r = 3(1 + cos 90 degrees) **M1**

(a) r = 3.00 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (0, 3.00) **A1**

(a) Answer: r = 3.00, (0, 3.00)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 6 at θ = 0 **A1**

(b) greatest distance from the pole = 6 **B1**

(b) Answer: Maximum r = 6 at θ = 0; greatest distance = 6.

Question 8

(a) right-hand loop: 0 ≤ θ ≤ π/4 or 7pi/4 ≤ θ < 2pi **B1**

(a) left-hand loop: 3pi/4 ≤ θ ≤ 5pi/4 **B1**

(a) Answer: Right loop: [0, π/4] union [7pi/4, 2pi); left loop: [3pi/4, 5pi/4].

(b) Area = (1/2) x integral from -π/4 to π/4 of 4cos(2theta) dtheta **M1**

(b) = 2 x integral from -π/4 to π/4 of cos(2theta) dtheta **M1**

(b) integrate: 2[sin(2theta)/2] = [sin(2theta)] **M1**

(b) substitute limits: sin(π/2) - sin(-π/2) = 1-(-1) = 2 **A1**

(b) Area = 2, cao **A1**

(b) Answer: Area of one loop = 2 (square units)

(c) set r = 0: cos(2theta) = 0 **M1**

(c) 2theta = +-π/2, giving θ = π/4 and θ = -π/4 **A1**

(c) interpretation: the lines θ = π/4 and θ = -π/4 are tangent to the curve at the pole, i.e. the loop touches the origin tangentially along these two directions **B1**

(c) Answer: θ = π/4 and θ = -π/4

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