1.6 Polar Coordinates: 30-minute Pack

Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has polar equation $r = 2(1 + \cos\theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 45$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 1 is 7 marks)

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2 The curve C has polar equation $r = 3(2 + \cos\theta)$, for $0 \leq \theta < 2pi$.

(a) Find the exact area enclosed by C. **(4)**

(Total for Question 2 is 4 marks)

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3 An Archimedean spiral has polar equation $r = 2\theta$, for $0 \leq \theta \leq 2pi$ (θ measured in radians).

(a) Complete the table of values of r. **(2)**

θ: 0, $\pi/2$, π, $3pi/2$, $2pi$

r: 0, ?, ?, ?, ?

(b) Find the exact area enclosed between the curve and the initial line for $0 \leq \theta \leq 2pi$ (the area swept out in one complete revolution). **(5)**

(c) A second spiral has equation $r = k\theta$ for the same interval $0 \leq \theta \leq 2pi$. Given that the area enclosed by this spiral (defined as in part (b)) is exactly twice the area found in part (b), find the value of k. **(3)**

(Total for Question 3 is 10 marks)

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4 A region is bounded by the curve $r = f(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ ($\alpha < \beta$), and the two half-lines $\theta = \alpha$ and $\theta = \beta$.

(a) By considering the region as the limit of a sum of thin circular sectors, prove that the area A of the region is given by A = (1/2) x integral from α to β of r^2 dtheta. **(5)**

(Total for Question 4 is 5 marks)

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Mark scheme · 1.6 Polar Coordinates: 30-minute Pack

Question 1

(a) r = 2(1 + cos 45 degrees) **M1**

(a) r = 3.41 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (2.41, 2.41) **A1**

(a) Answer: r = 3.41, (2.41, 2.41)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 4 at θ = 0 **A1**

(b) greatest distance from the pole = 4 **B1**

(b) Answer: Maximum r = 4 at θ = 0; greatest distance = 4.

Question 2

(a) area = (1/2) integral from 0 to 2pi of r^2 d θ **M1**

(a) expands 9(4 + 4cos θ + $\cos^2$ θ) **M1**

(a) integrates, using $\cos^2$ θ = (1 + cos 2theta)/2, and substitutes the limits **M1**

(a) 81pi/2 **A1**

(a) Answer: 81pi/2

Question 3

(a) r(π/2) = π and r(π) = 2pi **B1**

(a) r(3pi/2) = 3pi and r(2pi) = 4pi **B1**

(a) Answer: r = π, 2pi, 3pi, 4pi at θ = π/2, π, 3pi/2, 2pi respectively

(b) Area = (1/2) x integral from 0 to 2pi of $(2\text{theta})^2$ dtheta **M1**

(b) = 2 x integral from 0 to 2pi of θ^2 dtheta **M1**

(b) integrate: 2[θ^3/3] from 0 to 2pi **M1**

(b) substitute limits: 2 x $(2\text{pi})^3$/3 = 2 x 8pi^3/3 **A1**

(b) Area = 16pi^3/3, cao **A1**

(b) Answer: Area = 16pi^3/3 (approx 165 square units, 3 s.f.)

(c) new area = (k^2/2) x integral from 0 to 2pi of θ^2 dtheta = (k^2/2)(8pi^3/3) = $4k^2$ π^3/3 **M1**

(c) set $4k^2$ π^3/3 = 2 x 16pi^3/3, giving k^2 = 8 **M1**

(c) k = $2\sqrt{2}$, cao (positive root taken) **A1**

(c) Answer: k = $2\sqrt{2}$ (approx 2.83)

Question 4

(a) divide the region into n thin sectors, each subtending an angle delta(θ) = (β - α)/n at the pole **B1**

(a) the area of one such sector, of radius approximately r, is approximately (1/2) r^2 delta(θ), by the formula for the area of a circular sector **B1**

(a) the total area is approximately the sum over all n sectors: A is approximately σ (1/2) r^2 delta(θ), for θ from α to β **B1**

(a) as n tends to infinity, delta(θ) tends to 0, and the sum tends to the definite integral: A = (1/2) x integral from α to β of r^2 dtheta **B1**

(a) cso: the approximation error in each sector's area tends to zero faster than delta(θ) as delta(θ) -> 0, so the limit gives the exact area, establishing the result **B1**

(a) Answer: A = (1/2) x integral from α to β of r^2 dtheta

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