1.6 Polar Coordinates: Test Pack

Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

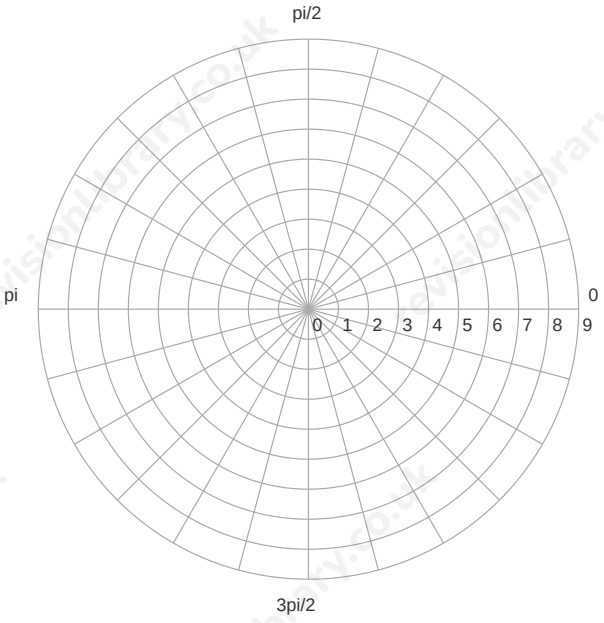
Answer ALL questions. Show all your working.

1 A curve has polar equation $r = 5 + 3\cos(\theta)$, for $-\pi < \theta \leq \pi$.

(a) State, with a reason, the symmetry of the curve. **(2)**

(b) Complete the table below, giving values to 3 significant figures where appropriate, and hence sketch the curve for $-\pi < \theta \leq \pi$. **(5)**

θ	0	π/6	π/3	π/2	2pi/3	5pi/6	π
r	8	?	6.5	5	?	?	2



(Total for Question 1 is 7 marks)

© Revision Library | revisionlibrary.co.uk

2 The curve C has polar equation $r = 4(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 60$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 2 is 7 marks)

---

3 The curve C has polar equation $r = 1 \sin(2 \theta)$, for $0 \leq \theta \leq \pi/2$. It forms one loop (petal) of a four-petal curve.

(a) Find the exact area enclosed by this loop. **(4)**

(Total for Question 3 is 4 marks)

---

4 A rose curve has polar equation r = 4 cos(3theta).

(a) State the number of petals of the curve and the maximum value of r. **(2)**

(b) One petal is traced as θ ranges from $-\pi/6$ to $\pi/6$. Show that the area of this petal is 4pi/3. **(5)**

(c) Hence find the total area enclosed by all three petals of the curve. **(2)**

(Total for Question 4 is 9 marks)

---

5 A limacon has polar equation $r = 1 + 2\cos(\theta)$, for $0 \leq \theta < 2pi$. Since the constant term (1) is less than the coefficient of $\cos(\theta)$ (2), the curve has an inner loop.

(a) Find the two values of θ, $0 \leq \theta < 2pi$, for which $r = 0$, and hence state the range of values of θ for which the curve traces the inner loop. **(3)**

(b) Find the exact area enclosed by the inner loop. **(6)**

(Total for Question 5 is 9 marks)

---

6 A circle C1 has polar equation $r = 3/2$. A cardioid C2 has polar equation $r = 1 + \cos(\theta)$, for $-\pi < \theta \leq \pi$. The two curves intersect and enclose a common region R, which is symmetrical about the initial line.

(a) Find the value of θ in the interval $0 \leq \theta \leq \pi$ at which C1 and C2 intersect. **(3)**

(b) Sketch C1 and C2 on the same diagram, for $-\pi < \theta \leq \pi$, showing clearly the region R that lies inside both curves. **(3)**

(c) Show that, for $0 \leq \theta \leq \pi/3$, the boundary of R nearer to the pole is C1, and that for $\pi/3 \leq \theta \leq \pi$, the boundary of R nearer to the pole is C2. **(2)**

(d) Hence show that the area of R is $7pi/4 - (9\sqrt{3})/8$. **(8)**

(Total for Question 6 is 16 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.6 Polar Coordinates: Test Pack

Question 1

(a) the curve is symmetrical about the initial line (θ = 0) **B1**

(a) valid reason: cos(-θ) = cos(θ), so r(θ) = r(-θ) for all θ **B1**

(a) Answer: Symmetrical about the initial line, since r is an even function of θ

(b) substitute θ = π/6, 2pi/3, 5pi/6 into r = 5 + 3cos(θ) **M1**

(b) r(π/6) = 5 + (3√3)/2 = 7.60 (3 s.f.) **A1**

(b) r(2pi/3) = 5 + 3cos(2pi/3) = 3.5 (3.50 also accepted) **A1**

(b) r(5pi/6) = 5 - (3√3)/2 = 2.40 (3 s.f.) **A1**

(b) smooth closed curve symmetrical about the initial line, with a single dimple (no inner loop, since 5 > 3), max r = 8 at θ = 0 and min r = 2 at θ = π **B1**

(b) Answer: r(π/6) = 7.60, r(2pi/3) = 3.5, r(5pi/6) = 2.40 (all 3 s.f. where relevant); dimpled limacon, symmetric about the initial line, ranging from r=8 at θ=0 to r=2 at θ=π

Question 2

(a) r = 4(1 + cos 60 degrees) **M1**

(a) r = 6.00 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (3.00, 5.20) **A1**

(a) Answer: r = 6.00, (3.00, 5.20)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 8 at θ = 0 **A1**

(b) greatest distance from the pole = 8 **B1**

(b) Answer: Maximum r = 8 at θ = 0; greatest distance = 8.

Question 3

(a) area = (1/2) integral of r^2 d θ **M1**

(a) uses $\sin^2(2\text{theta}) = (1 - \cos 4\text{theta})/2$ **M1**

(a) integrates and substitutes the limits 0 and π/2 **M1**

(a) π/8 **A1**

(a) Answer: π/8

Question 4

(a) 3 petals (the coefficient of θ, 3, is odd, so the curve has 3 petals) **B1**

(a) maximum r = 4 **B1**

(a) Answer: 3 petals, maximum r = 4

(b) Area = (1/2) x integral from -π/6 to π/6 of $16\cos^2(3\text{theta})$ dtheta **M1**

(b) use $\cos^2(3\text{theta}) = (1+\cos(6\text{theta}))/2$ to write the integrand as 4(1+cos(6theta)) **M1**

(b) integrate: 4[θ + sin(6theta)/6] **M1**

(b) substitute limits: 4[(π/6 + 0) - (-π/6 + 0)] = 4 x π/3 **A1**

(b) Area = 4pi/3, cso (answer printed) **A1**

(b) Answer: Area of one petal = 4pi/3

(c) total area = 3 x (4pi/3), using that the 3 petals do not overlap and the curve retraces the same petals over π ≤ θ < 2pi (since n is odd) **M1**

(c) total area = 4pi, cao **A1**

(c) Answer: Total area = 4pi (approx 12.6 square units)

Question 5

(a) set 1 + 2cos(θ) = 0, giving cos(θ) = -1/2 **M1**

(a) θ = 2pi/3 and θ = 4pi/3, cao **A1**

(a) the inner loop is traced for 2pi/3 ≤ θ ≤ 4pi/3, oe **B1**

(a) Answer: θ = 2pi/3, 4pi/3; inner loop traced for 2pi/3 ≤ θ ≤ 4pi/3

(b) Area = (1/2) x integral from 2pi/3 to 4pi/3 of $(1+2\cos(\theta))^2$ dtheta **M1**

(b) expand and use $\cos^2(\theta)=(1+\cos(2\text{theta}))/2$ to write integrand as 3 + 4cos(θ) + 2cos(2theta) **M1**

(b) integrate: 3theta + 4sin(θ) + sin(2theta) **M1**

(b) value at θ=4pi/3: 4pi - 2√3 + √3/2 = 4pi - (3√3)/2 **A1**

(b) value at θ=2pi/3: 2pi + 2√3 - √3/2 = 2pi + (3√3)/2 **A1**

(b) Area = (1/2)(2pi - 3√3) = π - (3√3)/2, cao **A1**

(b) Answer: Area of inner loop = π - (3√3)/2 (approx 0.544 square units, 3 s.f.)

Question 6

(a) set 3/2 = 1 + cos(θ) **M1**

(a) cos(θ) = 1/2 **A1**

(a) θ = π/3, cao **A1**

(a) Answer: θ = π/3 (and, by symmetry, θ = -π/3 in the full domain)

(b) circle C1 drawn correctly: centre the pole, radius 3/2 **B1**

(b) cardioid C2 drawn correctly: cusp at the pole (θ=π), maximum point (2,0) at θ=0, symmetrical about the initial line **B1**

(b) region R correctly identified and shaded as the overlap of the two curves, with the intersection points at θ = +-π/3 marked **B1**

(b) Answer: Diagram: circle radius 1.5 and cardioid with cusp at the pole, overlapping region R shaded between the two intersection points

(c) test θ=0: C1 gives r=1.5, C2 gives r=2, so C1 (the smaller value) is nearer the pole on 0≤θ≤π/3 **B1**

(c) test θ=π/2: C1 gives r=1.5, C2 gives r=1, so C2 (the smaller value) is nearer the pole on π/3≤θ≤π **B1**

(c) Answer: C1 is the inner boundary for 0≤θ≤π/3; C2 is the inner boundary for π/3≤θ≤π

(d) use symmetry to write Area = 2 x [(1/2) x integral from 0 to π/3 of $(3/2)^2$ dtheta + (1/2) x integral from π/3 to π of $(1+\cos(\theta))^2$ dtheta] **M1**

(d) expand $(1+\cos(\theta))^2 = 1 + 2\cos(\theta) + \cos^2(\theta)$ **M1**

(d) use $\cos^2(\theta) = (1+\cos(2\text{theta}))/2$ to write the integrand as 3/2 + 2cos(θ) + (1/2)cos(2theta) **M1**

(d) first integral: (1/2) x integral from 0 to π/3 of (9/4) dtheta = 3pi/8 **A1**

(d) integrate the second integrand: (3/2)θ + 2sin(θ) + (1/4)sin(2theta) **M1**

(d) second integral evaluates to π/2 - (9√3)/16 (oe unsimplified form accepted) **A1**

(d) combine: Area = 2 x [3pi/8 + π/2 - (9√3)/16] = 2 x [7pi/8 - (9√3)/16] **M1**

(d) Area = 7pi/4 - (9√3)/8, cso (answer printed) **A1**

(d) Answer: Area of R = 7pi/4 - (9√3)/8 (approx 3.55 square units, 3 s.f.)

© Revision Library | revisionlibrary.co.uk