1.6 Polar Coordinates

Calculator allowed · about 220 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Throughout this question, polar coordinates (r, θ) are given relative to the initial line and the pole O, with $r \geq 0$ unless stated otherwise.

(a) The point A has Cartesian coordinates $(3, -3\sqrt{3})$. Find the polar coordinates of A, giving r exactly and θ in radians such that $-\pi < \theta \leq \pi$. **(3)**

(b) A curve C has polar equation $r = 6\cos(\theta)$, for $-\pi/2 \leq \theta \leq \pi/2$. Show that the Cartesian equation of C can be written as $(x - 3)^2 + y^2 = 9$, and describe C geometrically. **(4)**

(Total for Question 1 is 7 marks)

2 A curve C has polar equation $r = 8 \sin(\theta)$, for $0 \leq \theta \leq \pi$.

(a) Show that the Cartesian equation of C can be written as $x^2 + (y - 4)^2 = 16$. **(3)**

(b) Hence state the centre and radius of C. **(2)**

(c) Using $A = (1/2) \times \int_{\theta=0}^{\theta=\pi} r^2 \, d\theta$, show that the area enclosed by C is 16π, and verify this agrees with your answer to part (b). **(4)**

(Total for Question 2 is 9 marks)

3 The curve C has polar equation $r = 2(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 45$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 3 is 7 marks)

4 The curve C has polar equation $r = 3(2 + \cos \theta)$, for $0 \leq \theta < 2\pi$.

(a) Find the exact area enclosed by C. **(4)**

(Total for Question 4 is 4 marks)

5 The curve C has polar equation $r = 6(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 120$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 5 is 7 marks)

6 The curve C has polar equation $r = 1(2 + \cos\theta)$, for $0 \leq \theta < 2pi$.

(a) Find the exact area enclosed by C. **(4)**

(Total for Question 6 is 4 marks)

7 The curve C has polar equation $r = 7(1 + \cos\theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 60$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 7 is 7 marks)

8 The curve C has polar equation $r = 3\sin(2\theta)$, for $0 \leq \theta \leq \pi/2$. It forms one loop (petal) of a four-petal curve.

(a) Find the exact area enclosed by this loop. **(4)**

(Total for Question 8 is 4 marks)

9 The curve C has polar equation $r = 7(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 120$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 9 is 7 marks)

10 The curve C has polar equation $r = 2(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$. The region R is bounded by C, the initial line and the half line $\theta = \pi/2$.

(a) Find the exact area of R. **(4)**

(Total for Question 10 is 4 marks)

11 The curve C has polar equation $r = 3(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 90$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 11 is 7 marks)

12 The curve C has polar equation $r = 6 \cos \theta$, for $-\pi/2 \leq \theta \leq \pi/2$.

(a) Find a Cartesian equation of C. **(3)**

(b) Hence write the equation of C in the form $(x - p)^2 + y^2 = q$, and state the centre and radius of the circle. **(3)**

(Total for Question 12 is 6 marks)

13 The curve C has polar equation $r = 3(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 120$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 13 is 7 marks)

14 The curve C has polar equation $r = 3(1 + \cos \theta)$, for $0 \leq \theta \leq \pi$.

(a) Find the exact area of the region bounded by C and the initial line. **(4)**

(Total for Question 14 is 4 marks)

15 The curve C has polar equation $r = 2(1 + \cos \theta)$, for $-\pi < \theta \leq \pi$.

(a) Find the value of r when $\theta = 60$ degrees, and the Cartesian coordinates of this point, to 3 significant figures. **(4)**

(b) Find the maximum value of r and the value of θ at which it occurs, and state the greatest distance of the curve from the pole. **(3)**

(Total for Question 15 is 7 marks)

16 The curve C has polar equation $r = 3\text{theta}$, for $0 \leq \theta \leq \pi$.

(a) Find the exact area of the region bounded by C and the initial line. **(4)**

(Total for Question 16 is 4 marks)

17 The curve C has polar equation $r = 6 \sin(2\theta)$, for $0 \leq \theta \leq \pi/2$. It forms one loop (petal) of a four-petal curve.

(a) Find the exact area enclosed by this loop. **(4)**

(Total for Question 17 is 4 marks)

18 The curve C has polar equation $r = 6(1 + \cos \theta)$, for $0 \leq \theta \leq \pi$.

(a) Find the exact area of the region bounded by C and the initial line. **(4)**

(Total for Question 18 is 4 marks)

19 The curve C has polar equation $r = 5\text{theta}$, for $0 \leq \theta \leq \pi$.

(a) Find the exact area of the region bounded by C and the initial line. **(4)**

(Total for Question 19 is 4 marks)

20 The curve C has polar equation $r = 4 \cos \theta$, for $-\pi/2 \leq \theta \leq \pi/2$.

(a) Find a Cartesian equation of C. **(3)**

(b) Hence write the equation of C in the form $(x - p)^2 + y^2 = q$, and state the centre and radius of the circle. **(3)**

(Total for Question 20 is 6 marks)

21 The curve C has polar equation $r = 6\cos(2\theta)$, for $0 \leq \theta \leq \pi/4$. (This is half of one petal.)

(a) Find the exact value of r when $\theta = \pi/6$, and the Cartesian coordinates of the point, to 3 significant figures. **(4)**

(b) Find the value of θ, in the range $0 \leq \theta \leq \pi/4$, at which the curve meets the pole. **(2)**

(Total for Question 21 is 6 marks)

22 A cardioid C has polar equation $r = 3(1 - \cos(\theta))$, for $0 \leq \theta \leq 2pi$.

(a) State the value of r when $\theta = 0$ and when $\theta = \pi$, and state the symmetry of C, giving a reason. **(3)**

(b) Find the exact area enclosed by C. **(6)**

(Total for Question 22 is 9 marks)

23 An Archimedean spiral has polar equation $r = 2\theta$, for $0 \leq \theta \leq 2\pi$ (θ measured in radians).

(a) Complete the table of values of r. **(2)**

θ: 0, $\pi/2$, π, $3\pi/2$, 2π

r: 0, ?, ?, ?, ?

(b) Find the exact area enclosed between the curve and the initial line for $0 \leq \theta \leq 2\pi$ (the area swept out in one complete revolution). **(5)**

(c) A second spiral has equation $r = k\theta$ for the same interval $0 \leq \theta \leq 2\pi$. Given that the area enclosed by this spiral (defined as in part (b)) is exactly twice the area found in part (b), find the value of k. **(3)**

(Total for Question 23 is 10 marks)

24 The cardioid C, with polar equation $r = 3(1 - \cos(\theta))$ for $0 \leq \theta < 2\pi$, is the curve from Question 22. Let $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

(a) Show that $dy/d\theta = 3\cos(\theta) - 3\cos(2\theta)$. **(3)**

(b) Hence find the exact coordinates of the two points on C, other than the pole, at which the tangent is parallel to the initial line. **(7)**

(Total for Question 24 is 10 marks)

25 A lemniscate has polar equation $r^2 = 4\cos(2\theta)$.

(a) State the two ranges of values of θ, for $0 \leq \theta < 2\pi$, for which the curve is defined (i.e. $r^2 \geq 0$), describing the right-hand and left-hand loops. **(2)**

(b) Find the exact area enclosed by one loop of the curve. **(5)**

(c) Find the values of θ at which the curve has a tangent at the pole for the right-hand loop, and briefly interpret this result. **(3)**

(Total for Question 25 is 10 marks)

26 Using the curve C from Question 1(b), with $r = 6\cos(\theta)$ for $-\pi/2 \leq \theta \leq \pi/2$, let $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

(a) Show that $x = 6\cos^2(\theta)$ and $y = 3\sin(2\theta)$, and hence find $dx/d\theta$ and $dy/d\theta$ in terms of θ. **(4)**

(b) Find the coordinates of the points on C at which (i) the tangent is parallel to the initial line, (ii) the tangent is perpendicular to the initial line. **(6)**

(Total for Question 26 is 10 marks)

27 A region is bounded by the curve $r = f(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ ($\alpha < \beta$), and the two half-lines $\theta = \alpha$ and $\theta = \beta$.

(a) By considering the region as the limit of a sum of thin circular sectors, prove that the area A of the region is given by A = (1/2) x integral from α to β of r^2 dtheta. **(5)**

(Total for Question 27 is 5 marks)

Mark scheme · 1.6 Polar Coordinates

Question 1

(a) $r^2 = 3^2 + (-3\sqrt{3})^2 = 9 + 27 = 36$ leading to $r = 6$ **M1**

(a) $\tan(\theta) = y/x = -\sqrt{3}$, reference angle $\pi/3$ **M1**

(a) $\theta = -\pi/3$ (fourth quadrant, since $x > 0$, $y < 0$), cao **A1**

(a) Answer: (r, θ) = (6, -π/3)

(b) multiply both sides by r: $r^2 = 6r\cos(\theta)$ **M1**

(b) use $r^2 = x^2 + y^2$ and $r\cos(\theta) = x$ to obtain $x^2 + y^2 = 6x$ **M1**

(b) complete the square: $(x - 3)^2 + y^2 = 9$, cso (answer printed) **A1**

(b) C is a circle with centre (3, 0) and radius 3 **B1**

(b) Answer: $(x-3)^2 + y^2 = 9$; circle, centre (3,0), radius 3

Question 2

(a) multiply by r: $r^2 = 8r\sin(\theta)$ **M1**

(a) substitute $r^2 = x^2+y^2$ and $r\sin(\theta) = y$: $x^2 + y^2 = 8y$ **M1**

(a) complete the square: $x^2 + (y-4)^2 = 16$, cso (answer printed) **A1**

(a) Answer: $x^2 + (y-4)^2 = 16$

(b) centre (0, 4) **B1**

(b) radius 4 **B1**

(b) Answer: Centre (0,4), radius 4

(c) Area = (1/2) x integral from 0 to π of $(8\sin(\theta))^2$ dtheta = 32 x integral from 0 to π of $\sin^2(\theta)$ dtheta **M1**

(c) use $\sin^2(\theta) = (1 - \cos(2\text{theta}))/2$ to write the integrand as $16(1-\cos(2\text{theta}))$ **M1**

(c) integrate: $16[\theta - \sin(2\text{theta})/2]$ evaluated from 0 to π = 16pi, cso **A1**

(c) verification: this matches π x radius^2 = π x 4^2 = 16pi from part (b) **B1**

(c) Answer: Area = 16pi

Question 3

(a) r = 2(1 + cos 45 degrees) **M1**

(a) r = 3.41 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (2.41, 2.41) **A1**

(a) Answer: r = 3.41, (2.41, 2.41)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 4 at θ = 0 **A1**

(b) greatest distance from the pole = 4 **B1**

(b) Answer: Maximum r = 4 at θ = 0; greatest distance = 4.

Question 4

(a) area = (1/2) integral from 0 to 2pi of r^2 d θ **M1**

(a) expands $9(4 + 4\cos\theta + \cos^2\theta)$ **M1**

(a) integrates, using $\cos^2\theta = (1 + \cos 2\text{theta})/2$, and substitutes the limits **M1**

(a) 81pi/2 **A1**

(a) Answer: 81pi/2

Question 5

(a) r = 6(1 + cos 120 degrees) **M1**

(a) r = 3.00 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (-1.50, 2.60) **A1**

(a) Answer: r = 3.00, (-1.50, 2.60)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 12 at θ = 0 **A1**

(b) greatest distance from the pole = 12 **B1**

(b) Answer: Maximum r = 12 at θ = 0; greatest distance = 12.

Question 6

(a) area = (1/2) integral from 0 to 2pi of r^2 d θ **M1**

(a) expands $1(4 + 4\cos\theta + \cos^2\theta)$ **M1**

(a) integrates, using $\cos^2\theta = (1 + \cos 2\text{theta})/2$, and substitutes the limits **M1**

(a) 9pi/2 **A1**

(a) Answer: 9pi/2

Question 7

(a) r = 7(1 + cos 60 degrees) **M1**

(a) r = 10.5 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (5.25, 9.09) **A1**

(a) Answer: r = 10.5, (5.25, 9.09)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 14 at θ = 0 **A1**

(b) greatest distance from the pole = 14 **B1**

(b) Answer: Maximum r = 14 at θ = 0; greatest distance = 14.

Question 8

(a) area = (1/2) integral of r^2 d θ **M1**

(a) uses $\sin^2(2\text{theta}) = (1 - \cos 4\text{theta})/2$ **M1**

(a) integrates and substitutes the limits 0 and π/2 **M1**

(a) 9pi/8 **A1**

(a) Answer: 9pi/8

Question 9

(a) r = 7(1 + cos 120 degrees) **M1**

(a) r = 3.50 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (-1.75, 3.03) **A1**

(a) Answer: r = 3.50, (-1.75, 3.03)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 14 at θ = 0 **A1**

(b) greatest distance from the pole = 14 **B1**

(b) Answer: Maximum r = 14 at θ = 0; greatest distance = 14.

Question 10

(a) area = (1/2) integral from 0 to π/2 of r^2 d θ **M1**

(a) uses $\cos^2\theta = (1 + \cos 2\text{theta})/2$ and integrates **M1**

(a) substitutes the limits 0 and π/2 **M1**

(a) 4(3pi/8 + 1) **A1**

(a) Answer: 4(3pi/8 + 1)

Question 11

(a) r = 3(1 + cos 90 degrees) **M1**

(a) r = 3.00 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (0, 3.00) **A1**

(a) Answer: r = 3.00, (0, 3.00)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 6 at θ = 0 **A1**

(b) greatest distance from the pole = 6 **B1**

(b) Answer: Maximum r = 6 at θ = 0; greatest distance = 6.

Question 12

(a) multiplies by r: $r^2 = 6 r \cos \theta$ **M1**

(a) uses $r^2 = x^2 + y^2$ and $r \cos \theta = x$ **M1**

(a) $x^2 + y^2 = 6x$ **A1**

(a) Answer: $x^2 + y^2 = 6x$

(b) completes the square **M1**

(b) $(x - 3)^2 + y^2 = 9$ **A1**

(b) centre (3, 0), radius 3 **A1**

(b) Answer: $(x - 3)^2 + y^2 = 9$; centre (3, 0), radius 3

Question 13

(a) r = 3(1 + cos 120 degrees) **M1**

(a) r = 1.50 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (-0.750, 1.30) **A1**

(a) Answer: r = 1.50, (-0.750, 1.30)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 6 at θ = 0 **A1**

(b) greatest distance from the pole = 6 **B1**

(b) Answer: Maximum r = 6 at θ = 0; greatest distance = 6.

Question 14

(a) area = (1/2) integral of r^2 d θ **M1**

(a) expands $9(1 + 2\cos \theta + \cos^2 \theta)$ and uses $\cos^2 \theta = (1 + \cos 2\text{theta})/2$ **M1**

(a) integrates: $(1/2)9[3\text{theta}/2 + 2 \sin \theta + \sin(2\text{theta})/4]$ **M1**

(a) 27pi/4 **A1**

(a) Answer: 27pi/4

Question 15

(a) r = 2(1 + cos 60 degrees) **M1**

(a) r = 3.00 **A1**

(a) x = r cos θ and y = r sin θ **M1**

(a) (1.50, 2.60) **A1**

(a) Answer: r = 3.00, (1.50, 2.60)

(b) cos θ has a maximum of 1 **M1**

(b) r_{max} = 4 at θ = 0 **A1**

(b) greatest distance from the pole = 4 **B1**

(b) Answer: Maximum r = 4 at θ = 0; greatest distance = 4.

Question 16

(a) area = (1/2) integral of r^2 d θ **M1**

(a) integrates $(1/2)9\text{theta}^2$ to $3/2\text{theta}^3$ **M1**

(a) substitutes the limit π and 0 **A1**

(a) $9\text{pi}^3/6$ **A1**

(a) Answer: $9\text{pi}^3/6$

Question 17

(a) area = (1/2) integral of r^2 d θ **M1**

(a) uses $\sin^2(2\text{theta}) = (1 - \cos 4\text{theta})/2$ **M1**

(a) integrates and substitutes the limits 0 and π/2 **M1**

(a) 36pi/8 **A1**

(a) Answer: 36pi/8

Question 18

(a) area = (1/2) integral of r^2 d θ **M1**

(a) expands $36(1 + 2\cos \theta + \cos^2 \theta)$ and uses $\cos^2 \theta = (1 + \cos 2\text{theta})/2$ **M1**

(a) integrates: $(1/2)36[3\text{theta}/2 + 2 \sin \theta + \sin(2\text{theta})/4]$ **M1**

(a) 27pi **A1**

(a) Answer: 27pi

Question 19

(a) area = (1/2) integral of r^2 d θ **M1**

(a) integrates $(1/2)25\text{theta}^2$ to $25/6\text{theta}^3$ **M1**

(a) substitutes the limit π and 0 **A1**

(a) $25\text{pi}^3/6$ **A1**

(a) Answer: $25\text{pi}^3/6$

Question 20

(a) multiplies by r: $r^2 = 4 r \cos \theta$ **M1**

(a) uses $r^2 = x^2 + y^2$ and $r \cos \theta = x$ **M1**

(a) $x^2 + y^2 = 4x$ **A1**

(a) Answer: $x^2 + y^2 = 4x$

(b) completes the square **M1**

(b) $(x - 2)^2 + y^2 = 4$ **A1**

(b) centre (2, 0), radius 2 **A1**

(b) Answer: $(x - 2)^2 + y^2 = 4$; centre (2, 0), radius 2

Question 21

(a) r = 6 cos(π/3) **M1**

(a) r = 3 **A1**

(a) x = r cos(π/6), y = r sin(π/6) **M1**

(a) (2.60, 1.50) **A1**

(a) Answer: r = 3, (2.60, 1.50)

(b) sets r = 0: cos(2 θ) = 0 **M1**

(b) 2 θ = π/2, so θ = π/4 **A1**

(b) Answer: θ = π/4

Question 22

(a) r = 0 when θ = 0 (C has a cusp at the pole) **B1**

(a) r = 6 when θ = π (the point furthest from the pole) **B1**

(a) symmetrical about the initial line, since cos(-θ) = cos(θ) **B1**

(a) Answer: r(0) = 0, r(π) = 6; symmetrical about the initial line

(b) Area = (1/2) x integral from 0 to 2pi of $[3(1-\cos(\theta))]^2$ dtheta **M1**

(b) expand: $(1-\cos(\theta))^2 = 1 - 2\cos(\theta) + \cos^2(\theta)$ **M1**

(b) use $\cos^2(\theta) = (1+\cos(2\text{theta}))/2$ throughout the integrand **M1**

(b) integral from 0 to 2pi of 1 dtheta = 2pi, integral of cos(θ) dtheta = 0, integral of $\cos^2(\theta)$ dtheta = π **A1**

(b) combine: Area = (9/2)(2pi - 0 + π) = (9/2)(3pi) **M1**

(b) Area = 27pi/2, cao **A1**

(b) Answer: Area = 27pi/2 (= 13.5pi, approx 42.4 square units)

Question 23

(a) r(π/2) = π and r(π) = 2pi **B1**

(a) r(3pi/2) = 3pi and r(2pi) = 4pi **B1**

(a) Answer: r = π, 2pi, 3pi, 4pi at θ = π/2, π, 3pi/2, 2pi respectively

(b) Area = (1/2) x integral from 0 to 2pi of $(2\text{theta})^2$ dtheta **M1**

(b) = 2 x integral from 0 to 2pi of θ^2 dtheta **M1**

(b) integrate: $2[\theta^3/3]$ from 0 to 2pi **M1**

(b) substitute limits: 2 x $(2\text{pi})^3/3$ = 2 x $8\text{pi}^3/3$ **A1**

(b) Area = $16\text{pi}^3/3$, cao **A1**

(b) Answer: Area = $16\text{pi}^3/3$ (approx 165 square units, 3 s.f.)

(c) new area = $(k^2/2)$ x integral from 0 to 2pi of θ^2 dtheta = $(k^2/2)(8\text{pi}^3/3) = 4k^2\,\pi^3/3$ **M1**

(c) set $4k^2\,\pi^3/3 = 2$ x $16\text{pi}^3/3$, giving $k^2 = 8$ **M1**

(c) $k = 2\sqrt{2}$, cao (positive root taken) **A1**

(c) Answer: $k = 2\sqrt{2}$ (approx 2.83)

Question 24

(a) write y = 3(1-cos(θ))sin(θ) = 3sin(θ) - 3sin(θ)cos(θ) **M1**

(a) use sin(θ)cos(θ) = (1/2)sin(2theta), so y = 3sin(θ) - (3/2)sin(2theta) **M1**

(a) differentiate: dy/dtheta = 3cos(θ) - 3cos(2theta), cso **A1**

(a) Answer: dy/dtheta = 3cos(θ) - 3cos(2theta)

(b) set dy/dtheta = 0: cos(θ) = cos(2theta) **M1**

(b) use $\cos(2\text{theta}) = 2\cos^2(\theta) - 1$ to form $2\cos^2(\theta) - \cos(\theta) - 1 = 0$ **M1**

(b) factorise (2cos(θ)+1)(cos(θ)-1) = 0, giving cos(θ) = -1/2 (rejecting cos(θ)=1, θ=0, the cusp at the pole) **A1**

(b) θ = 2pi/3 and θ = 4pi/3 **A1**

(b) find r = 3(1-(-1/2)) = 9/2 at both values **M1**

(b) point 1: (x,y) = $(-9/4, 9\sqrt{3}/4)$, cao **A1**

(b) point 2: (x,y) = $(-9/4, -9\sqrt{3}/4)$, ft from point 1 (mirror image under the symmetry of part (a) reasoning) **B1ft**

(b) Answer: $(-9/4, 9\sqrt{3}/4)$ and $(-9/4, -9\sqrt{3}/4)$

Question 25

(a) right-hand loop: 0 ≤ θ ≤ π/4 or 7pi/4 ≤ θ < 2pi **B1**

(a) left-hand loop: 3pi/4 ≤ θ ≤ 5pi/4 **B1**

(a) Answer: Right loop: [0, π/4] union [7pi/4, 2pi); left loop: [3pi/4, 5pi/4].

(b) Area = (1/2) x integral from -π/4 to π/4 of 4cos(2theta) dtheta **M1**

(b) = 2 x integral from -π/4 to π/4 of cos(2theta) dtheta **M1**

(b) integrate: 2[sin(2theta)/2] = [sin(2theta)] **M1**

(b) substitute limits: sin(π/2) - sin(-π/2) = 1-(-1) = 2 **A1**

(b) Area = 2, cao **A1**

(b) Answer: Area of one loop = 2 (square units)

(c) set r = 0: cos(2theta) = 0 **M1**

(c) 2theta = +-π/2, giving θ = π/4 and θ = -π/4 **A1**

(c) interpretation: the lines θ = π/4 and θ = -π/4 are tangent to the curve at the pole, i.e. the loop touches the origin tangentially along these two directions **B1**

(c) Answer: θ = π/4 and θ = -π/4

Question 26

(a) x = r cos(θ) = 6cos(θ)cos(θ) = $6\cos^2(\theta)$; y = r sin(θ) = 6cos(θ)sin(θ) = 3sin(2theta) **B1**

(a) differentiate x: dx/dtheta = -12sin(θ)cos(θ) **M1**

(a) dx/dtheta = -6sin(2theta), oe **A1**

(a) dy/dtheta = 6cos(2theta) **A1**

(a) Answer: dx/dtheta = -6sin(2theta), dy/dtheta = 6cos(2theta)

(b) (i) set dy/dtheta = 0: cos(2theta) = 0, giving θ = π/4 or θ = -π/4 **M1**

(b) (i) find r = $6\cos(\pi/4) = 3\sqrt{2}$ at each, then x = r cos(θ) = 3, y = r sin(θ) = ±3 **M1**

(b) (i) points (3, 3) and (3, -3), cao **A1**

(b) (ii) set dx/dtheta = 0: sin(2theta) = 0, giving θ = 0 or θ = +-π/2 **M1**

(b) (ii) at θ = 0, r = 6, giving the point (6, 0) **A1**

(b) (ii) at θ = +-π/2, r = 0, giving the pole (0, 0); both points have a vertical tangent **A1**

(b) Answer: (i) (3,3) and (3,-3); (ii) (6,0) and the pole (0,0)

Question 27

(a) divide the region into n thin sectors, each subtending an angle delta(θ) = (β - α)/n at the pole **B1**

(a) the area of one such sector, of radius approximately r, is approximately (1/2) r^2 delta(θ), by the formula for the area of a circular sector **B1**

(a) the total area is approximately the sum over all n sectors: A is approximately σ (1/2) r^2 delta(θ), for θ from α to β **B1**

(a) as n tends to infinity, delta(θ) tends to 0, and the sum tends to the definite integral: A = (1/2) x integral from α to β of r^2 dtheta **B1**

(a) cso: the approximation error in each sector's area tends to zero faster than delta(θ) as delta(θ) -> 0, so the limit gives the exact area, establishing the result **B1**

(a) Answer: A = (1/2) x integral from α to β of r^2 dtheta