1.7 Hyperbolic Functions: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 It is given that $\sinh x = 48/55$, where x is real.

(a) Use a hyperbolic identity to find the exact value of $\cosh x$, and hence $\tanh x$. **(4)**

(b) Find the exact value of x, giving your answer as a natural logarithm. **(3)**

(c) Find the exact value of $\sinh 2x$ and of $\cosh 2x$. **(3)**

(Total for Question 1 is 10 marks)

---

2 Solve the equation $21\cosh x - 15\sinh x = 15$, giving your answers as exact logarithms.

(a) Show that the equation can be written as $6e^{2x} - 30e^{x} + 36 = 0$. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 2 is 7 marks)

---

3 This question is about the inverse hyperbolic function arsinh x, defined for all real x.

(a) Let $y = \text{arsinh } x$, so that $x = \sinh y$. By writing this equation in exponential form, show that $e^{2y} - 2x\,e^{y} - 1 = 0$, and hence show that $\text{arsinh } x = \ln(x + \sqrt{x^2+1})$ for all real x. **(5)**

(b) Hence find, in the form $\ln(a + \sqrt{b})$ where a and b are integers, the exact value of arsinh(2). **(2)**

(c) Solve the equation $\text{arsinh}(x) = \ln 3$, giving your answer as an exact fraction. **(3)**

(Total for Question 3 is 10 marks)

© Revision Library | revisionlibrary.co.uk

4 This question is about differentiating hyperbolic and inverse hyperbolic functions.

(a) Differentiate $y = \sinh(3x^2 - 1)$ with respect to x. **(2)**

(b) Find dy/dx where $y = \ln(\cosh 2x)$, giving your answer in terms of tanh 2x. **(3)**

(c) Given that $y = \text{arcosh}(3x)$, for $x > 1/3$, show by implicit differentiation that $dy/dx = 3/\sqrt{9x^2-1}$. **(3)**

(d) Find the equation of the tangent to the curve $y = \text{arsinh}(x/2)$ at the point where $x = 0$, giving your answer in the form $y = mx$. **(4)**

(Total for Question 4 is 12 marks)

---

5 This question is about integrating functions involving hyperbolic functions.

(a) Find, in terms of x, the integral: integral of $\sinh(4x - 1)\,dx$. **(2)**

(b) Find, in terms of arsinh, the integral: integral of $1/\sqrt{x^2+9}\,dx$. **(3)**

(c) Using the substitution $x = 4 \sinh t$, show that integral of $1/\sqrt{x^2+16}\,dx = \text{arsinh}(x/4) + c$. **(5)**

(d) Hence evaluate exactly the integral from $x = 0$ to $x = 4$ of $1/\sqrt{x^2+16}\,dx$, giving your answer as a single natural logarithm. **(2)**

(Total for Question 5 is 12 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.7 Hyperbolic Functions: 1-hour Pack

Question 1

(a) uses $\cosh^2 x - \sinh^2 x = 1$	**M1**
(a) $\cosh^2 x = 1 + 2304/3025 = 5329/3025$	**A1**
(a) $\cosh x = 73/55$ (positive root, since $\cosh x \geq 1$)	**A1**
(a) $\tanh x = 48/73$	**A1**
(a) Answer: $\cosh x = 73/55$, $\tanh x = 48/73$	
(b) uses the definition $\sinh x = (e^x - e^{-x})/2$ or the logarithmic form of arsinh	**M1**
(b) $e^x = \sinh x + \cosh x = 11/5$	**A1**
(b) $x = \ln(11/5)$	**A1**
(b) Answer: $x = \ln(11/5)$	
(c) uses $\sinh 2x = 2 \sinh x \cosh x$ and $\cosh 2x = 1 + 2\sinh^2 x$	**M1**
(c) $\sinh 2x = 7008/3025$	**A1**
(c) $\cosh 2x = 7633/3025$	**A1**
(c) Answer: $\sinh 2x = 7008/3025$, $\cosh 2x = 7633/3025$	

Question 2

(a) writes $\cosh x = (e^x + e^{-x})/2$ and $\sinh x = (e^x - e^{-x})/2$	**M1**
(a) multiplies through by e^x and collects terms	**M1**
(a) $6e^{2x} - 30e^x + 36 = 0$ (cso)	**A1**
(a) Answer: $6e^{2x} - 30e^x + 36 = 0$	
(b) treats as a quadratic in e^x and factorises	**M1**
(b) $e^x = 2$ or $e^x = 3$	**A1**
(b) takes natural logs	**M1**
(b) $x = \ln 2$ and $x = \ln 3$	**A1**
(b) Answer: $x = \ln 2$, $x = \ln 3$	

Question 3

(a) write $x = \sinh y = (e^y - e^{-y})/2$	**M1**
(a) multiply by $2e^y$ and rearrange to $e^{2y} - 2x e^y - 1 = 0$	**M1**
(a) solve this quadratic in e^y using the quadratic formula: $e^y = x \pm \sqrt{x^2+1}$	**M1**
(a) reject the negative root since $e^y > 0$ but $x - \sqrt{x^2+1} < 0$ for all x	**A1**
(a) hence $y = \ln(x + \sqrt{x^2+1})$, i.e. $\text{arsinh}\, x = \ln(x+\sqrt{x^2+1})$, cso	**A1**
(a) Answer: $\text{arsinh}\, x = \ln(x + \sqrt{x^2+1})$	
(b) substitute $x = 2$ into the formula from part (a)	**M1**
(b) $\text{arsinh}(2) = \ln(2 + \sqrt{5})$, cao	**A1**
(b) Answer: $\text{arsinh}(2) = \ln(2 + \sqrt{5})$	
(c) take sinh of both sides: $x = \sinh(\ln 3) = (3 - 1/3)/2$	**M1**
(c) simplify $(3 - 1/3)/2 = (8/3)/2$	**M1**
(c) $x = 4/3$, cao	**A1**
(c) Answer: $x = 4/3$	

Question 4

(a) apply the chain rule to obtain the form $k \cdot x \cdot \cosh(3x^2-1)$	**M1**
(a) $dy/dx = 6x \cosh(3x^2-1)$, cao	**A1**
(a) Answer: $dy/dx = 6x \cosh(3x^2 - 1)$	
(b) use $d/dx[\ln u] = u'/u$ with $u = \cosh 2x$, $u' = 2\sinh 2x$	**M1**
(b) $dy/dx = 2\sinh 2x / \cosh 2x$	**A1**
(b) $dy/dx = 2\tanh 2x$, cao	**A1**
(b) Answer: $dy/dx = 2\tanh 2x$	
(c) write $\cosh y = 3x$ and differentiate implicitly: $\sinh y \cdot dy/dx = 3$	**M1**
(c) use $\sinh y = \sqrt{\cosh^2(y) - 1} = \sqrt{9x^2-1}$, taking the positive root since $y > 0$	**M1**
(c) $dy/dx = 3/\sqrt{9x^2-1}$, cso	**A1**
(c) Answer: $dy/dx = 3/\sqrt{9x^2 - 1}$	
(d) identify the point $(0, 0)$ since $\text{arsinh}(0) = 0$	**B1**
(d) differentiate using $d/dx[\text{arsinh}(u)] = u'/\sqrt{u^2+1}$ with $u = x/2$, $u' = 1/2$	**M1**
(d) dy/dx at $x = 0$ is $(1/2)/\sqrt{0+1} = 1/2$	**A1**
(d) tangent: $y = (1/2)x$, oe (e.g. $2y = x$), cao	**A1**
(d) Answer: $y = (1/2)x$	

Question 5

(a) form $k \cosh(4x-1)$	**M1**
(a) $(1/4)\cosh(4x-1) + c$, cao	**A1**
(a) Answer: $(1/4)\cosh(4x-1) + c$	
(b) quote the standard result integral of $1/\sqrt{x^2+a^2}\ dx = \text{arsinh}(x/a) + c$	**B1**
(b) identify $a^2 = 9$, so $a = 3$	**M1**
(b) $\text{arsinh}(x/3) + c$, cao	**A1**
(b) Answer: $\text{arsinh}(x/3) + c$	
(c) substitute $x = 4\sinh t$, $dx = 4\cosh t\ dt$	**M1**
(c) use $1 + \sinh^2(t) = \cosh^2(t)$ to write $\sqrt{x^2+16} = \sqrt{16\sinh^2(t)+16} = 4\cosh t$	**M1**
(c) integral becomes integral of $(4\cosh t)/(4\cosh t)\ dt =$ integral of $1\ dt$	**A1**
(c) $= t + c$, and $t = \text{arsinh}(x/4)$ since $x/4 = \sinh t$	**A1**
(c) integral of $1/\sqrt{x^2+16}\ dx = \text{arsinh}(x/4) + c$, cso (answer printed)	**A1**
(c) Answer: $\text{arsinh}(x/4) + c$	
(d) substitute limits: $\text{arsinh}(1) - \text{arsinh}(0)$	**M1**
(d) $= \ln(1 + \sqrt{2})$, cao	**A1**
(d) Answer: $\ln(1 + \sqrt{2})$	

© Revision Library | revisionlibrary.co.uk