1.7 Hyperbolic Functions: 30-minute Pack

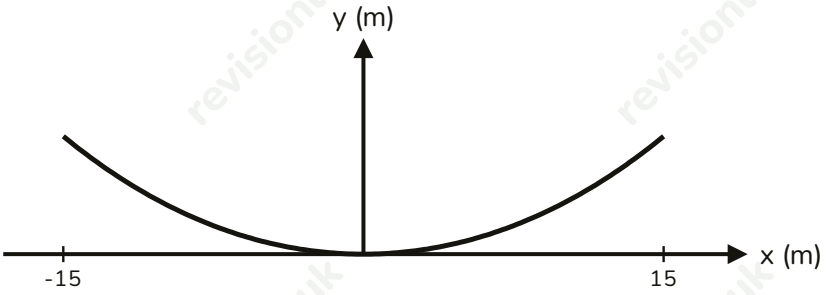
EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 An engineer, Priya, models a cable hanging between two supports of equal height as a catenary with equation $y = 20\cosh(x/20) - 20$, for $-15 \leq x \leq 15$, where x and y are measured in metres and $x = 0$ at the lowest point of the cable.



(a) Find the sag of the cable, that is, the value of y at $x = 15$, giving your answer in metres to 3 significant figures. **(3)**

(b) Find dy/dx and hence find the gradient of the cable at $x = 15$, giving your answer to 3 significant figures. **(3)**

(c) Using the arc length formula L = integral from x=-15 to x=15 of $\sqrt{1 + (dy/dx)^2}$ dx, and the identity $1 + \sinh^2(t) = \cosh^2(t)$, show that the total length of cable between the two supports is 32.9 m, correct to 3 significant figures. **(5)**

(Total for Question 1 is 11 marks)

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2 Solve the equation $32\cosh x - 28\sinh x = 16$, giving your answers as exact logarithms.

(a) Show that the equation can be written as $4e^{2x} - 32e^{x} + 60 = 0$. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 2 is 7 marks)

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3 This question is about integrating hyperbolic and related functions.

(a) Find the exact value of the integral from 0 to 9/4 of $1/\sqrt{x^2 + 9}$ dx, giving your answer as a natural logarithm. **(4)**

(Total for Question 3 is 4 marks)

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4 This question is about integration leading to the logarithmic form of artanh.

(a) Using the standard result integral of $1/(a^2-x^2)$ dx = (1/a) artanh(x/a) + c and the logarithmic form artanh(u) = (1/2) ln((1+u)/(1-u)), show that, for $-3 < x < 3$, integral of $1/(9-x^2)$ dx = (1/6) ln((3+x)/(3-x)) + c. **(4)**

(b) Hence find the exact value of the integral from x = 0 to x = 1 of $1/(9-x^2)$ dx, giving your answer in the form (1/6) ln k, stating the value of k. **(3)**

(Total for Question 4 is 7 marks)

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Mark scheme · 1.7 Hyperbolic Functions: 30-minute Pack

Question 1

(a) substitute x = 15 into y = 20cosh(x/20) - 20 **M1**

(a) evaluate cosh(0.75) using a calculator **M1**

(a) y = 5.89 m (awrt 5.89), cao **A1**

(a) Answer: 5.89 m

(b) differentiate: dy/dx = 20 . (1/20) sinh(x/20) = sinh(x/20) **M1**

(b) substitute x = 15 and evaluate sinh(0.75) **M1**

(b) gradient = 0.822 (awrt 0.822), cao **A1**

(b) Answer: dy/dx = sinh(x/20); at x = 15, gradient is awrt 0.822

(c) state L = integral from -15 to 15 of $\sqrt{1+\sinh^2(x/20)}$ dx, using their dy/dx from part (b) **M1**

(c) use $1 + \sinh^2(x/20) = \cosh^2(x/20)$ to simplify the integrand to cosh(x/20) **M1**

(c) integrate: integral of cosh(x/20) dx = 20 sinh(x/20) **M1**

(c) evaluate limits: 20sinh(0.75) - 20sinh(-0.75) = 40 sinh(0.75) **A1**

(c) L = 40(0.82232) = 32.9 m (awrt 32.9), cso **A1**

(c) Answer: L = 32.9 m

Question 2

(a) writes cosh x = $(e^x + e^{-x})/2$ and sinh x = $(e^x - e^{-x})/2$ **M1**

(a) multiplies through by e^x and collects terms **M1**

(a) $4e^{2x} - 32e^x + 60 = 0$ (cso) **A1**

(a) Answer: $4e^{2x} - 32e^x + 60 = 0$

(b) treats as a quadratic in e^x and factorises **M1**

(b) $e^x = 3$ or $e^x = 5$ **A1**

(b) takes natural logs **M1**

(b) x = ln 3 and x = ln 5 **A1**

(b) Answer: x = ln 3, x = ln 5

Question 3

(a) uses the standard integral: arsinh(x/3) **M1**

(a) arsinh(3/4) = $\ln(3/4 + \sqrt{1 + 9/16})$ **A1**

(a) $\sqrt{1 + 9/16} = 5/4$ **A1**

(a) ln 2 **A1**

(a) Answer: ln 2

Question 4

(a) apply the standard result with a = 3: (1/3) artanh(x/3) + c **M1**

(a) substitute artanh(x/3) = (1/2) ln((1+x/3)/(1-x/3)) **M1**

(a) simplify (1/3)(1/2)ln((1+x/3)/(1-x/3)) to (1/6) ln((3+x)/(3-x)) **A1**

(a) integral of $1/(9-x^2)$ dx = (1/6) ln((3+x)/(3-x)) + c, cso (answer printed) **A1**

(a) Answer: (1/6) ln((3+x)/(3-x)) + c

(b) substitute limits: (1/6)ln(4/2) - (1/6)ln(3/3) **M1**

(b) simplify to (1/6) ln 2, since ln(3/3) = ln 1 = 0 **A1**

(b) k = 2, cao **A1**

(b) Answer: (1/6) ln 2, so k = 2

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