1.7 Hyperbolic Functions: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question concerns the exponential definitions of the hyperbolic functions.

(a) State, using exponential functions, the definitions of cosh x and sinh x. **(2)**

(b) Given that cosh x = 2 and x > 0, use the identity $\cosh^2(x) - \sinh^2(x) = 1$ to find the exact value of sinh x, and hence write down the exact value of tanh x. **(3)**

(c) Find the exact value of e^x. **(2)**

(Total for Question 1 is 7 marks)

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2 Throughout this question you must derive each result from the exponential definitions of cosh x and sinh x, showing each line of algebra clearly.

(a) Show that $\cosh^2(x) - \sinh^2(x) = 1$ for all real x. **(3)**

(b) Show that sinh 2x = 2 sinh x cosh x. **(3)**

(c) Hence, or otherwise, show that $\cosh 2x = 1 + 2\sinh^2(x)$. **(3)**

(Total for Question 2 is 9 marks)

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3 This question is about integrating hyperbolic and related functions.

(a) Find the exact value of the integral from 0 to 5/3 of $1/\sqrt{x^2 + 16}$ dx, giving your answer as a natural logarithm. **(4)**

(Total for Question 3 is 4 marks)

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4 The curve C has equation y = 10 cosh x - 5 sinh x.

(a) Find dy/dx and show that C has a stationary point at x = (1/2) ln(3). **(4)**

(b) Find the exact y coordinate of the stationary point, and determine its nature. **(3)**

(Total for Question 4 is 7 marks)

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5 Consider the equation $2\cosh x - \sinh x = 2$.

(a) Show that the equation $2\cosh x - \sinh x = 2$ can be written as $e^{2x} - 4e^{x} + 3 = 0$. **(4)**

(b) Hence solve the equation $2\cosh x - \sinh x = 2$, giving your answer(s) exactly. **(4)**

(Total for Question 5 is 8 marks)

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6 The curve C has parametric equations $x = 5\cosh t$, $y = 3\sinh t$, for t in R.

(a) Show that the Cartesian equation of C is $x^2/25 - y^2/9 = 1$. **(3)**

(b) Find dy/dx in terms of t, in its simplest form. **(3)**

(c) The point P on C has parameter $t = p$. Show that the equation of the tangent to C at P can be written as $(x\cosh p)/5 - (y\sinh p)/3 = 1$. **(5)**

(Total for Question 6 is 11 marks)

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Mark scheme · 1.7 Hyperbolic Functions: Test Pack

Question 1

(a) cosh x = $(e^x + e^{-x})/2$ oe **B1**

(a) sinh x = $(e^x - e^{-x})/2$ oe **B1**

(a) Answer: cosh x = $(e^x + e^{-x})/2$, sinh x = $(e^x - e^{-x})/2$

(b) $\sinh^2(x) = \cosh^2(x) - 1 = 2^2 - 1 = 3$ **M1**

(b) sinh x = $\sqrt{3}$ (positive root taken since x > 0), cao **A1**

(b) tanh x = $\sqrt{3}/2$, ft from their sinh x **B1**

(b) Answer: sinh x = $\sqrt{3}$, tanh x = $\sqrt{3}/2$

(c) e^x = cosh x + sinh x **M1**

(c) $e^x = 2 + \sqrt{3}$, cao **A1**

(c) Answer: $e^x = 2 + \sqrt{3}$

Question 2

(a) expand $\cosh^2(x) = (e^x+e^{-x})^2/4$ and $\sinh^2(x) = (e^x-e^{-x})^2/4$ **M1**

(a) correct expansions $(e^{2x}+2+e^{-2x})/4$ and $(e^{2x}-2+e^{-2x})/4$ **A1**

(a) subtract to obtain 4/4 = 1, cso (answer printed) **A1**

(a) Answer: $\cosh^2(x) - \sinh^2(x) = 1$

(b) write 2 sinh x cosh x = 2 . $[(e^x-e^{-x})/2]$. $[(e^x+e^{-x})/2]$ **M1**

(b) simplify to $(e^{2x} - e^{-2x})/2$ **A1**

(b) identify this as sinh 2x by the exponential definition, cso **A1**

(b) Answer: sinh 2x = 2 sinh x cosh x

(c) write $\cosh 2x = (e^{2x}+e^{-2x})/2$ by definition **M1**

(c) expand $2\sinh^2(x) = (e^{2x} - 2 + e^{-2x})/2$ **M1**

(c) $1 + 2\sinh^2(x)$ simplifies to $(e^{2x}+e^{-2x})/2 = \cosh 2x$, cso (answer printed) **A1**

(c) Answer: $\cosh 2x = 1 + 2\sinh^2(x)$

Question 3

(a) uses the standard integral: arsinh(x/4) **M1**

(a) $\text{arsinh}(5/12) = \ln(5/12 + \sqrt{1 + 25/144})$ **A1**

(a) $\sqrt{1 + 25/144} = 13/12$ **A1**

(a) ln(3/2) **A1**

(a) Answer: ln(3/2)

Question 4

(a) dy/dx = 10 sinh x - 5 cosh x **M1**

(a) sets dy/dx = 0: tanh x = 1/2 **M1**

(a) x = artanh(1/2) = (1/2) ln((1 + 1/2)/(1 - 1/2)) **A1**

(a) = (1/2) ln(3) (cso) **A1**

(a) Answer: x = (1/2) ln(3)

(b) substitutes: cosh x = $10/\sqrt{75}$ etc, or writes y = $\sqrt{75}$ cosh(x - c) **M1**

(b) y = $5\sqrt{3}$ **A1**

(b) $d^2y/dx^2 = y > 0$ at the stationary point, so a minimum **B1**

(b) Answer: y = $5\sqrt{3}$ (minimum)

Question 5

(a) write 2cosh x - sinh x in exponential form **M1**

(a) simplify to $e^x + 3e^{-x} = 4$ oe **A1**

(a) multiply through by e^x **M1**

(a) $e^{2x} - 4e^x + 3 = 0$, cso (answer printed) **A1**

(a) Answer: $e^{2x} - 4e^x + 3 = 0$

(b) let $u = e^x$ to form $u^2 - 4u + 3 = 0$ **M1**

(b) factorise (u-1)(u-3) = 0, or use the quadratic formula **M1**

(b) u = 1 or u = 3, i.e. $e^x = 1$ or $e^x = 3$ **A1**

(b) x = 0 and x = ln 3, both required, cao **A1**

(b) Answer: x = 0 or x = ln 3

Question 6

(a) write cosh t = x/5 and sinh t = y/3 **M1**

(a) use $\cosh^2(t) - \sinh^2(t) = 1$ **M1**

(a) $x^2/25 - y^2/9 = 1$, cso (answer printed) **A1**

(a) Answer: $x^2/25 - y^2/9 = 1$

(b) differentiate: dx/dt = 5 sinh t, dy/dt = 3 cosh t **M1**

(b) use dy/dx = (dy/dt) / (dx/dt) **M1**

(b) dy/dx = (3/5) coth t, oe (e.g. 3cosh t / (5 sinh t)), cao **A1**

(b) Answer: dy/dx = (3/5) coth t

(c) write the tangent at P(5cosh p, 3sinh p) using gradient (3/5)coth p: y - 3sinh p = (3/5)coth p (x - 5cosh p) **M1**

(c) multiply throughout by 5 sinh p to clear fractions **M1**

(c) obtain $5y \sinh p - 15\sinh^2(p) = 3x \cosh p - 15\cosh^2(p)$ **A1**

(c) use $\cosh^2(p) - \sinh^2(p) = 1$ to rearrange to 3x cosh p - 5y sinh p = 15 **M1**

(c) divide by 15 to obtain (x cosh p)/5 - (y sinh p)/3 = 1, cso (answer printed) **A1**

(c) Answer: (x cosh p)/5 - (y sinh p)/3 = 1

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