1.8 Differential Equations: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Find the solution of the differential equation $dy/dx + 2y = e^x$ for which $y = 0$ when $x = 0$.

(a) Find the integrating factor and the general solution. **(4)**

(b) Hence find the particular solution with $y = 0$ when $x = 0$. **(3)**

(Total for Question 1 is 7 marks)

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2 A curve satisfies the second order differential equation $d^2y/dx^2 - 6(dy/dx) + 9y = 0$.

(a) Show that the auxiliary equation has a repeated root, and state its value. **(2)**

(b) Write down the general solution of the differential equation. **(1)**

(c) Given that $y = 2$ and $dy/dx = 1$ when $x = 0$, find the particular solution of the differential equation. **(5)**

(Total for Question 2 is 8 marks)

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3 A curve satisfies the second order differential equation $d^2y/dx^2 + 4(dy/dx) + 13y = 0$.

(a) Show that the roots of the auxiliary equation are $-2 + 3i$ and $-2 - 3i$. **(3)**

(b) State the general solution of the differential equation. **(2)**

(c) Given that $y = 0$ and $dy/dx = 6$ when $x = 0$, find the particular solution of the differential equation. **(4)**

(d) Describe the long-term behaviour of y as x tends to infinity, giving a reason for your answer. **(2)**

(Total for Question 3 is 11 marks)

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4 It is given that $y = e^{-x}(3\cos 2x - \sin 2x)$.

(a) Show that y satisfies the differential equation $d^2y/dx^2 + 2(dy/dx) + 5y = 0$. **(5)**

(b) State the general solution of the differential equation $d^2y/dx^2 + 2(dy/dx) + 5y = 0$, justifying the form of your answer by reference to the auxiliary equation. **(3)**

(c) Find, to 3 significant figures, the two values of x in the interval $0 \leq x \leq \pi$ for which $y = 0$, where y is the function given at the start of the question. **(4)**

(Total for Question 4 is 12 marks)

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5 A curve satisfies the second order differential equation $d^2y/dx^2 - 3(dy/dx) - 4y = 8x^2 - 2$.

(a) Find the complementary function of the differential equation. **(3)**

(b) By trying a particular integral of the form $y = ax^2 + bx + c$, find a particular integral for the differential equation. **(5)**

(c) Hence write down the general solution of the differential equation. **(1)**

(d) Given that $y = 0$ and $dy/dx = 0$ when $x = 0$, find the particular solution of the differential equation. **(4)**

(Total for Question 5 is 13 marks)

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Mark scheme · 1.8 Differential Equations: 1-hour Pack

Question 1

(a) integrating factor e^{2x} **B1**

(a) $d/dx(y\ e^{2x}) = e^{3x}$ **M1**

(a) $y\ e^{2x} = e^{3x}/3 + C$ **A1**

(a) $y = e^{x}/3 + Ce^{-2x}$ **A1**

(a) Answer: $y = e^{x}/3 + Ce^{-2x}$

(b) substitutes x = 0, y = 0: $0 = 1/3 + C$ **M1**

(b) $C = -1/3$ **A1**

(b) $y = e^{x}/3 + (-1/3)e^{-2x}$ **A1**

(b) Answer: $y = e^{x}/3 + (-1/3)e^{-2x}$

Question 2

(a) auxiliary equation $m^2-6m+9=0$ written as $(m-3)^2=0$, or discriminant = 36-36 = 0 shown **M1**

(a) repeated root m = 3, cso **A1**

(a) Answer: Repeated root m = 3

(b) $y = (A + Bx)e^{3x}$, ft from part (a) **B1**

(b) Answer: $y = (A + Bx)e^{3x}$

(c) differentiates using the product rule **M1**

(c) correct $dy/dx = (3A+B+3Bx)e^{3x}$ **A1**

(c) substitutes x=0 into y and dy/dx to form equations A=2 and 3A+B=1 **M1**

(c) A = 2, B = -5 (both required) **A1**

(c) $y = (2-5x)e^{3x}$, cao **A1**

(c) Answer: $y = (2 - 5x)e^{3x}$

Question 3

(a) writes auxiliary equation $m^2+4m+13=0$ **M1**

(a) uses the quadratic formula or completes the square: $m = (-4 \pm \sqrt{16-52})/2$ **M1**

(a) m = -2 ± 3i, cso (both roots required, answer printed) **A1**

(a) Answer: m = -2 + 3i or m = -2 - 3i

(b) recognises complex roots p ± qi give solutions of the form $e^{px}(A\cos(qx) + B\sin(qx))$ **M1**

(b) $y = e^{-2x}(A\cos3x + B\sin3x)$, cao **A1**

(b) Answer: $y = e^{-2x}(A\cos3x + B\sin3x)$

(c) differentiates the general solution using the product rule **M1**

(c) substitutes x=0 into y and dy/dx to form equations A=0 and -2A+3B=6 **M1**

(c) A = 0, B = 2 (both required) **A1**

(c) $y = 2e^{-2x}\sin3x$, cao **A1**

(c) Answer: $y = 2e^{-2x}\sin3x$

(d) y oscillates with decreasing amplitude, tending to 0 as x tends to infinity **B1**

(d) correct reason: e^{-2x} tends to 0 while sin3x remains bounded between -1 and 1 **B1**

(d) Answer: y tends to 0 as x tends to infinity (a damped oscillation)

Question 4

(a) differentiates y using the product rule **M1**

(a) correct $dy/dx = -5e^{-x}(\cos2x + \sin2x)$ **A1**

(a) differentiates again using the product rule **M1**

(a) correct $d^2y/dx^2 = e^{-x}(-5\cos2x + 15\sin2x)$ **A1**

(a) substitutes into $d^2y/dx^2 + 2(dy/dx) + 5y$ and shows the cos2x and sin2x coefficients both sum to 0, cso **A1**

(a) Answer: $d^2y/dx^2 + 2(dy/dx) + 5y = 0$, as required

(b) solves auxiliary equation $m^2+2m+5=0$ using the quadratic formula **M1**

(b) $m = -1 \pm 2i$ **A1**

(b) general solution $y = e^{-x}(C\cos2x + D\sin2x)$, oe **B1**

(b) Answer: $y = e^{-x}(C\cos2x + D\sin2x)$

(c) sets $3\cos2x - \sin2x = 0$ and rearranges to $\tan2x = 3$ **M1**

(c) finds two values of 2x in [0, 2pi] satisfying $\tan2x = 3$ **M1**

(c) x = 0.625 (3sf) **A1**

(c) x = 2.20 (3sf) **A1**

(c) Answer: x = 0.625 or x = 2.20 (3sf)

Question 5

(a) auxiliary equation $m^2-3m-4=0$ **M1**

(a) factorises to $(m-4)(m+1)=0$, giving m=4, m=-1 **A1**

(a) complementary function $y = Ae^{4x} + Be^{-x}$, ft **A1**

(a) Answer: CF: $y = Ae^{4x} + Be^{-x}$

(b) differentiates trial twice: $dy/dx=2ax+b$, $d^2y/dx^2=2a$ **M1**

(b) substitutes into the differential equation and compares coefficients of x^2, x and constants **M1**

(b) a = -2 **A1**

(b) b = 3 **A1**

(b) c = -11/4 **A1**

(b) Answer: PI: $y = -2x^2 + 3x - 11/4$

(c) $y = Ae^{4x} + Be^{-x} - 2x^2 + 3x - 11/4$, ft **B1**

(c) Answer: $y = Ae^{4x} + Be^{-x} - 2x^2 + 3x - 11/4$

(d) substitutes x=0, y=0 into the general solution to form A+B-11/4=0 **M1**

(d) differentiates and substitutes x=0, dy/dx=0 to form 4A-B+3=0 **M1**

(d) A = -1/20 (or B = 14/5) **A1**

(d) A = -1/20 and B = 14/5, both stated in $y = -1/20\ e^{4x} + 14/5\ e^{-x} - 2x^2 + 3x - 11/4$, cao **A1**

(d) Answer: $y = -1/20\ e^{4x} + 14/5\ e^{-x} - 2x^2 + 3x - 11/4$

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