1.8 Differential Equations: 30-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 In an electrical circuit, the charge Q coulombs on a capacitor at time t seconds satisfies the differential equation $dQ/dt + 5Q = 15$, with $Q = 0$ when $t = 0$.

(a) Show that the integrating factor for this differential equation is e^{5t}. **(2)**

(b) Hence show that $Q = 3(1 - e^{-5t})$. **(4)**

(c) State the maximum charge that the capacitor can hold, explaining your reasoning. **(2)**

(d) Find, to 3 significant figures, the time taken for the charge to reach 2.9 coulombs. **(3)**

(Total for Question 1 is 11 marks)

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2 Find the general solution of $x\,dy/dx + 2y = x^3$, for $x > 0$, giving y in terms of x.

(a) Write the equation in the form $dy/dx + P(x)y = Q(x)$ and find the integrating factor. **(2)**

(b) Hence find the general solution. **(3)**

(Total for Question 2 is 5 marks)

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3 Find the general solution of $x\,dy/dx + y = x^3$, for $x > 0$, giving y in terms of x.

(a) Write the equation in the form $dy/dx + P(x)y = Q(x)$ and find the integrating factor. **(2)**

(b) Hence find the general solution. **(3)**

(Total for Question 3 is 5 marks)

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4 Solve the differential equation $d^2y/dx^2 - 8\,dy/dx + 16y = 0$, given that $y = 5$ and $dy/dx = 4$ when $x = 0$.

(a) Show that the auxiliary equation has a repeated root, and write down the general solution. **(3)**

(b) Find the particular solution. **(4)**

(Total for Question 4 is 7 marks)

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Mark scheme · 1.8 Differential Equations: 30-minute Pack

Question 1

(a) attempts integrating factor $e^{\text{integral of 5 dt}}$	**M1**
(a) IF = e^{5t}, cso (answer printed)	**A1**
(a) Answer: Integrating factor = e^{5t}	
(b) multiplies through by e^{5t} and integrates: Q e^{5t} = integral of 15 e^{5t} dt	**M1**
(b) Q e^{5t} = 3 e^{5t} + c, oe	**A1**
(b) uses Q = 0 when t = 0 to find c = -3	**M1**
(b) Q = 3 - 3e^{-5t} = 3(1 - e^{-5t}), cso (answer printed)	**A1**
(b) Answer: Q = 3(1 - e^{-5t})	
(c) maximum charge = 3 coulombs	**B1**
(c) correct reason: as t tends to infinity, e^{-5t} tends to 0, so Q tends to 3	**B1**
(c) Answer: Maximum charge = 3 coulombs	
(d) sets up 2.9 = 3(1-e^{-5t})	**M1**
(d) rearranges to e^{-5t} = 1/30 and takes natural logs, t = -(1/5)ln(1/30)	**M1**
(d) t = 0.680 s (3sf), awrt	**A1**
(d) Answer: t = 0.680 s (3sf)	

Question 2

(a) divides by x: dy/dx + (2/x)y = x^2	**M1**
(a) integrating factor exp(integral of 2/x dx) = x^2	**B1**
(a) Answer: IF = x^2	
(b) d/dx(x^2y) = x^2 x x^2 = x^4	**M1**
(b) x^2y = x^5/5 + C	**A1**
(b) y = x^3/5 + C x^{-2}	**A1**
(b) Answer: y = x^3/5 + C x^{-2}	

Question 3

(a) divides by x: dy/dx + (1/x)y = x^2	**M1**
(a) integrating factor exp(integral of 1/x dx) = x	**B1**
(a) Answer: IF = x	
(b) d/dx(xy) = x x x^2 = x^3	**M1**
(b) xy = x^4/4 + C	**A1**
(b) y = x^3/4 + C x^{-1}	**A1**
(b) Answer: y = x^3/4 + C x^{-1}	

Question 4

(a) auxiliary equation $(m - 4)^2$ = 0	**M1**
(a) repeated root m = 4	**A1**
(a) y = (A + Bx)e^{4x}	**A1**
(a) Answer: y = (A + Bx)e^{4x}	
(b) y(0) = A = 5	**M1**
(b) y' = (B + 4(A + Bx))e^{4x}, so y'(0) = B + 4A = 4	**M1**
(b) B = -16	**A1**
(b) y = (5 - 16x)e^{4x}	**A1**
(b) Answer: y = (5 - 16x)e^{4x}	

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