1.8 Differential Equations: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A curve satisfies the first order linear differential equation $dy/dx + (2/x)y = x^2$, for $x > 0$.

(a) Show that the integrating factor for this differential equation is x^2. **(2)**

(b) Hence find the general solution of the differential equation, giving y in terms of x. **(4)**

(c) Given that $y = 3/2$ when $x = 1$, find the particular solution of the differential equation. **(3)**

(Total for Question 1 is 9 marks)

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2 Solve the differential equation $d^2y/dx^2 - 5\,dy/dx + 6y = 0$, given that $y = 4$ and $dy/dx = 5$ when $x = 0$.

(a) Write down the auxiliary equation and find the general solution. **(3)**

(b) Find the particular solution. **(4)**

(Total for Question 2 is 7 marks)

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3 A particle P moves on the x axis so that its displacement x metres from the origin at time t seconds satisfies $d^2x/dt^2 = -25x$. When $t = 0$, $x = 4$ and $dx/dt = 4$.

(a) Show that the general solution is $x = A\cos(5t) + B\sin(5t)$, and find the values of A and B. **(4)**

(b) Find the period of the motion and the amplitude, to 3 significant figures. **(3)**

(c) Find the maximum speed of P, to 3 significant figures. **(2)**

(Total for Question 3 is 9 marks)

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4 A curve satisfies the second order differential equation $d^2y/dx^2 - 5(dy/dx) + 6y = 0$.

(a) Write down the auxiliary equation and hence find its roots. **(2)**

(b) Write down the general solution of the differential equation. **(2)**

(c) Given that $y = 1$ and $dy/dx = 4$ when $x = 0$, find the particular solution of the differential equation. **(4)**

(Total for Question 4 is 8 marks)

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5 A particle of mass 0.5 kg is attached to a spring and moves through a resistive medium. The displacement x metres from equilibrium at time t seconds satisfies the differential equation $0.5(d^2x/dt^2) + 2(dx/dt) + 10x = 0$.

(a) Show that the differential equation can be written as $d^2x/dt^2 + 4(dx/dt) + 20x = 0$. **(1)**

(b) Find the general solution for x in terms of t. **(4)**

(c) The particle is released from rest at a displacement of 0.5 m from equilibrium. Find x in terms of t. **(4)**

(d) State, with a reason, what happens to the displacement of the particle as t becomes large. **(2)**

(e) Determine whether the motion described is oscillatory, critically damped or overdamped, justifying your answer with reference to the roots of the auxiliary equation. **(2)**

(Total for Question 5 is 13 marks)

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Mark scheme · 1.8 Differential Equations: Test Pack

Question 1

(a) attempts integrating factor $e^{\text{integral of } 2/x\ dx} = e^{2 \ln x}$ **M1**

(a) correctly simplifies $e^{2 \ln x} = e^{\ln x^2} = x^2$, cso (answer printed) **A1**

(a) Answer: Integrating factor = x^2

(b) multiplies through by x^2 to obtain $d/dx(x^2 y) = x^2 . x^2$ **M1**

(b) correct right hand side x^4 **A1**

(b) integrates both sides: $x^2 y = (x^5)/5 + C$ **M1**

(b) $y = (x^3)/5 + C/x^2$, oe **A1**

(b) Answer: $y = x^3/5 + C/x^2$

(c) substitutes x = 1, y = 3/2 into the general solution **M1**

(c) C = 13/10, oe **A1**

(c) $y = x^3/5 + 13/(10x^2)$, cao **A1**

(c) Answer: $y = x^3/5 + 13/(10x^2)$

Question 2

(a) auxiliary equation $m^2 - 5m + 6 = 0$ **M1**

(a) m = 2 and m = 3 **A1**

(a) $y = Ae^{2x} + Be^{3x}$ **A1**

(a) Answer: $y = Ae^{2x} + Be^{3x}$

(b) uses y(0) to get A + B = 4 **M1**

(b) uses y'(0): 2A + 3B = 5 **M1**

(b) A = 7, B = -3 **A1**

(b) $y = (7)e^{2x} + (-3)e^{3x}$ **A1**

(b) Answer: $y = (7)e^{2x} + (-3)e^{3x}$

Question 3

(a) auxiliary equation $m^2 + 25 = 0$, so $m = \pm 5i$ **M1**

(a) x = A cos(5t) + B sin(5t) **A1**

(a) uses x(0) = 4 and x'(0) = 5B = 4 **M1**

(a) A = 4, B = 4/5 **A1**

(a) Answer: A = 4, B = 4/5

(b) period = 2pi/5 = 1.26 s **B1**

(b) amplitude = $\sqrt{A^2 + B^2}$ **M1**

(b) amplitude = 4.08 m **A1**

(b) Answer: period 1.26 s, amplitude 4.08 m

(c) maximum speed = ω x amplitude **M1**

(c) 20.4 m/s **A1**

(c) Answer: 20.4 m/s

Question 4

(a) auxiliary equation $m^2 - 5m + 6 = 0$ factorised as (m-2)(m-3) = 0 **M1**

(a) m = 2, m = 3, cao (both required) **A1**

(a) Answer: m = 2, m = 3

(b) correct form $y = Ae^{2x} + Be^{3x}$ **B1**

(b) correct use of two arbitrary constants A and B, ft from part (a) **B1**

(b) Answer: $y = Ae^{2x} + Be^{3x}$

(c) differentiates general solution: $dy/dx = 2Ae^{2x} + 3Be^{3x}$ **M1**

(c) forms two simultaneous equations A+B=1 and 2A+3B=4 using the given conditions **M1**

(c) A = -1, B = 2 (both required) **A1**

(c) $y = -e^{2x} + 2e^{3x}$, cao **A1**

(c) Answer: $y = -e^{2x} + 2e^{3x}$

Question 5

(a) multiplies through by 2 to obtain $d^2x/dt^2+4(dx/dt)+20x=0$, cso (answer printed) **B1**

(a) Answer: $d^2x/dt^2 + 4(dx/dt) + 20x = 0$

(b) auxiliary equation $m^2+4m+20=0$ **M1**

(b) uses the quadratic formula: $m = (-4 \pm \sqrt{16-80})/2$ **M1**

(b) $m = -2 \pm 4i$ **A1**

(b) $x = e^{-2t}(A \cos 4t + B \sin 4t)$ **A1**

(b) Answer: $x = e^{-2t}(A \cos 4t + B \sin 4t)$

(c) uses x(0)=0.5 to obtain A=0.5 **M1**

(c) differentiates and uses dx/dt=0 at t=0 to form -2A+4B=0 **M1**

(c) B = 0.25 **A1**

(c) $x = e^{-2t}(0.5\cos 4t + 0.25\sin 4t)$, cao **A1**

(c) Answer: $x = e^{-2t}(0.5 \cos 4t + 0.25 \sin 4t)$

(d) x tends to 0 as t becomes large **B1**

(d) correct reason: e^{-2t} tends to 0 while the trigonometric factor remains bounded, so the oscillation dies away **B1**

(d) Answer: x tends to 0 as t becomes large

(e) the motion is (damped) oscillatory / underdamped **B1**

(e) correct justification: the auxiliary equation has complex (non-real) roots -2+-4i, so the solution is a decaying exponential multiplying a genuine oscillation, rather than a single or double real root as in critical or overdamping **B1**

(e) Answer: The motion is damped oscillatory (underdamped)

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