1.8 Differential Equations

EDEXCEL 9FM0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 In an electrical circuit, the charge Q coulombs on a capacitor at time t seconds satisfies the differential equation $dQ/dt + 5Q = 15$, with $Q = 0$ when $t = 0$.

(a) Show that the integrating factor for this differential equation is e^{5t}. **(2)**

(b) Hence show that $Q = 3(1 - e^{-5t})$. **(4)**

(c) State the maximum charge that the capacitor can hold, explaining your reasoning. **(2)**

(d) Find, to 3 significant figures, the time taken for the charge to reach 2.9 coulombs. **(3)**

(Total for Question 1 is 11 marks)

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2 Find the general solution of $x\,dy/dx + 2y = x^3$, for $x > 0$, giving y in terms of x.

(a) Write the equation in the form $dy/dx + P(x)y = Q(x)$ and find the integrating factor. **(2)**

(b) Hence find the general solution. **(3)**

(Total for Question 2 is 5 marks)

3 A particle P moves on the x axis so that its displacement x metres from the origin at time t seconds satisfies $d^2x/dt^2 = -25x$. When $t = 0$, $x = 2$ and $dx/dt = -3$.

(a) Show that the general solution is $x = A\cos(5t) + B\sin(5t)$, and find the values of A and B. **(4)**

(b) Find the period of the motion and the amplitude, to 3 significant figures. **(3)**

(c) Find the maximum speed of P, to 3 significant figures. **(2)**

(Total for Question 3 is 9 marks)

4 Find the solution of the differential equation $dy/dx + 2y = e^x$ for which $y = 0$ when $x = 0$.

(a) Find the integrating factor and the general solution. **(4)**

(b) Hence find the particular solution with $y = 0$ when $x = 0$. **(3)**

(Total for Question 4 is 7 marks)

5 Find the general solution of $x\,dy/dx + y = x^3$, for $x > 0$, giving y in terms of x.

(a) Write the equation in the form $dy/dx + P(x)y = Q(x)$ and find the integrating factor. **(2)**

(b) Hence find the general solution. **(3)**

(Total for Question 5 is 5 marks)

6 Solve the differential equation $d^2y/dx^2 + 4\,dy/dx + 20y = 0$, given that $y = 4$ and $dy/dx = 1$ when $x = 0$.

(a) Find the roots of the auxiliary equation and write down the general solution. **(3)**

(b) Find the particular solution. **(4)**

(Total for Question 6 is 7 marks)

7 Solve the differential equation $d^2y/dx^2 - 8\,dy/dx + 16y = 0$, given that $y = 5$ and $dy/dx = 4$ when $x = 0$.

(a) Show that the auxiliary equation has a repeated root, and write down the general solution. **(3)**

(b) Find the particular solution. **(4)**

(Total for Question 7 is 7 marks)

8 Find the solution of the differential equation $d^2y/dx^2 + 4\,dy/dx + 3y = -3e^{2x}$ for which $y = -2$ and $dy/dx = 1$ when $x = 0$.

(a) Find the complementary function. **(3)**

(b) Find a particular integral of the form $y = k\,e^{2x}$. **(3)**

(c) Hence find the particular solution satisfying the initial conditions. **(4)**

(Total for Question 8 is 10 marks)

9 Find the solution of the differential equation $d^2y/dx^2 + 4\,dy/dx + 3y = -4e^{2x}$ for which $y = 3$ and $dy/dx = -2$ when $x = 0$.

(a) Find the complementary function. **(3)**

(b) Find a particular integral of the form $y = k\,e^{2x}$. **(3)**

(c) Hence find the particular solution satisfying the initial conditions. **(4)**

(Total for Question 9 is 10 marks)

10 A curve satisfies the second order differential equation $d^2y/dx^2 - 6(dy/dx) + 9y = 0$.

(a) Show that the auxiliary equation has a repeated root, and state its value. **(2)**

(b) Write down the general solution of the differential equation. **(1)**

(c) Given that $y = 2$ and $dy/dx = 1$ when $x = 0$, find the particular solution of the differential equation. **(5)**

(Total for Question 10 is 8 marks)

11 A curve satisfies the second order differential equation $d^2y/dx^2 + 4(dy/dx) + 13y = 0$.

(a) Show that the roots of the auxiliary equation are $-2 + 3i$ and $-2 - 3i$. **(3)**

(b) State the general solution of the differential equation. **(2)**

(c) Given that $y = 0$ and $dy/dx = 6$ when $x = 0$, find the particular solution of the differential equation. **(4)**

(d) Describe the long-term behaviour of y as x tends to infinity, giving a reason for your answer. **(2)**

(Total for Question 11 is 11 marks)

12 It is given that $y = e^{-x}(3\cos 2x - \sin 2x)$.

(a) Show that y satisfies the differential equation $d^2y/dx^2 + 2(dy/dx) + 5y = 0$. **(5)**

(b) State the general solution of the differential equation $d^2y/dx^2 + 2(dy/dx) + 5y = 0$, justifying the form of your answer by reference to the auxiliary equation. **(3)**

(c) Find, to 3 significant figures, the two values of x in the interval $0 \leq x \leq \pi$ for which $y = 0$, where y is the function given at the start of the question. **(4)**

(Total for Question 12 is 12 marks)

13 A curve satisfies the second order differential equation $d^2y/dx^2 - 3(dy/dx) - 4y = 8x^2 - 2$.

(a) Find the complementary function of the differential equation. **(3)**

(b) By trying a particular integral of the form $y = ax^2 + bx + c$, find a particular integral for the differential equation. **(5)**

(c) Hence write down the general solution of the differential equation. **(1)**

(d) Given that $y = 0$ and $dy/dx = 0$ when $x = 0$, find the particular solution of the differential equation. **(4)**

(Total for Question 13 is 13 marks)

14 A curve satisfies the second order differential equation $d^2y/dx^2 - 4(dy/dx) + 3y = e^{3x}$.

(a) Show that m = 1 and m = 3 are roots of the auxiliary equation, hence state the complementary function. **(3)**

(b) Explain why a particular integral of the form $y = Ce^{3x}$ is not suitable for this differential equation, and state an appropriate alternative form to try. **(2)**

(c) Using your alternative form from part (b), find the particular integral. **(5)**

(d) Write down the general solution of the differential equation. **(1)**

(Total for Question 14 is 11 marks)

15 A curve satisfies the second order differential equation $d^2y/dx^2 + y = \sin 2x$.

(a) Find the complementary function of the differential equation. **(2)**

(b) By trying a particular integral of the form $y = P\cos 2x + Q\sin 2x$, find the values of P and Q. **(5)**

(c) Hence state the general solution of the differential equation. **(1)**

(d) Given that $y = 1$ and $dy/dx = 0$ when $x = 0$, find the particular solution of the differential equation. **(5)**

(Total for Question 15 is 13 marks)

16 The differential equation $x^2(d^2y/dx^2) + x(dy/dx) - 4y = 0$, for $x > 0$, is to be solved using the substitution $x = e^u$. You may use, without proof, that when $x = e^u$, $x(dy/dx) = dy/du$ and $x^2(d^2y/dx^2) = (d^2y/du^2) - (dy/du)$.

(a) Using the given substitution results, show that the differential equation transforms to $d^2y/du^2 - 4y = 0$. **(3)**

(b) Hence find y in terms of u, and show that the general solution can be written as $y = Ax^2 + B/x^2$. **(4)**

(c) Given that $y = 3$ and $dy/dx = -1$ when $x = 1$, find the particular solution of the differential equation. **(5)**

(Total for Question 16 is 12 marks)

17 Two variables x and y satisfy the simultaneous first order differential equations $dx/dt = x + 2y$ and $dy/dt = 3x + 2y$. Given that $x = 5$ and $y = 0$ when $t = 0$, answer the following.

(a) By eliminating y, show that x satisfies the differential equation $d^2x/dt^2 - 3(dx/dt) - 4x = 0$. **(4)**

(b) Hence find x in terms of t, given that $x = 5$ and $y = 0$ when $t = 0$. **(5)**

(c) Find y in terms of t. **(3)**

(Total for Question 17 is 12 marks)

18 A forced oscillator satisfies the differential equation $d^2x/dt^2 + 6(dx/dt) + 25x = 51 \cos t$, where x is displacement in metres and t is time in seconds.

(a) Find the roots of the auxiliary equation, and hence state the complementary function. **(3)**

(b) By trying a particular integral of the form $x = P \cos t + Q \sin t$, show that $P = 2$ and $Q = 0.5$. **(5)**

(c) Given that $x = 0$ and $dx/dt = 0$ when $t = 0$, find the particular solution of the differential equation. **(5)**

(d) State what is meant by the transient and steady state parts of the solution, identifying each part in your answer to part (c). **(3)**

(Total for Question 18 is 16 marks)

Mark scheme · 1.8 Differential Equations

Question 1

(a) attempts integrating factor $e^{\text{integral of 5 dt}}$ **M1**

(a) IF = e^{5t}, cso (answer printed) **A1**

(a) Answer: Integrating factor = e^{5t}

(b) multiplies through by e^{5t} and integrates: Q e^{5t} = integral of 15 e^{5t} dt **M1**

(b) Q e^{5t} = 3 e^{5t} + c, oe **A1**

(b) uses Q = 0 when t = 0 to find c = -3 **M1**

(b) Q = 3 - 3e^{-5t} = 3(1 - e^{-5t}), cso (answer printed) **A1**

(b) Answer: Q = 3(1 - e^{-5t})

(c) maximum charge = 3 coulombs **B1**

(c) correct reason: as t tends to infinity, e^{-5t} tends to 0, so Q tends to 3 **B1**

(c) Answer: Maximum charge = 3 coulombs

(d) sets up 2.9 = 3(1-e^{-5t}) **M1**

(d) rearranges to e^{-5t} = 1/30 and takes natural logs, t = -(1/5)ln(1/30) **M1**

(d) t = 0.680 s (3sf), awrt **A1**

(d) Answer: t = 0.680 s (3sf)

Question 2

(a) divides by x: dy/dx + (2/x)y = x^2 **M1**

(a) integrating factor exp(integral of 2/x dx) = x^2 **B1**

(a) Answer: IF = x^2

(b) d/dx(x^2y) = x^2 x x^2 = x^4 **M1**

(b) x^2y = x^5/5 + C **A1**

(b) y = x^3/5 + C x^{-2} **A1**

(b) Answer: y = x^3/5 + C x^{-2}

Question 3

(a) auxiliary equation m^2 + 25 = 0, so m = ±5i **M1**

(a) x = A cos(5t) + B sin(5t) **A1**

(a) uses x(0) = 2 and x'(0) = 5B = -3 **M1**

(a) A = 2, B = -3/5 **A1**

(a) Answer: A = 2, B = -3/5

(b) period = 2pi/5 = 1.26 s **B1**

(b) amplitude = $\sqrt{A^2 + B^2}$ **M1**

(b) amplitude = 2.09 m **A1**

(b) Answer: period 1.26 s, amplitude 2.09 m

(c) maximum speed = ω x amplitude **M1**

(c) 10.4 m/s **A1**

(c) Answer: 10.4 m/s

Question 4

(a) integrating factor e^{2x} **B1**

(a) d/dx(y e^{2x}) = e^{3x} **M1**

(a) y e^{2x} = e^{3x}/3 + C **A1**

(a) y = e^x/3 + Ce^{-2x} **A1**

(a) Answer: y = e^x/3 + Ce^{-2x}

(b) substitutes x = 0, y = 0: 0 = 1/3 + C **M1**

(b) C = -1/3 **A1**

(b) y = e^x/3 + (-1/3)e^{-2x} **A1**

(b) Answer: y = e^x/3 + (-1/3)e^{-2x}

Question 5

(a) divides by x: dy/dx + (1/x)y = x^2 **M1**

(a) integrating factor exp(integral of 1/x dx) = x **B1**

(a) Answer: IF = x

(b) d/dx(xy) = x x x^2 = x^3 **M1**

(b) xy = x^4/4 + C **A1**

(b) y = x^3/4 + C x^{-1} **A1**

(b) Answer: y = x^3/4 + C x^{-1}

Question 6

(a) solves the auxiliary equation using the quadratic formula **M1**

(a) m = -2 ± 4i **A1**

(a) y = e^{-2x}(A cos(4x) + B sin(4x)) **A1**

(a) Answer: y = e^{-2x}(A cos(4x) + B sin(4x))

(b) y(0) = A = 4 **M1**

(b) y'(0) = -2A + 4B = 1 **M1**

(b) B = 9/4 **A1**

(b) y = e^{-2x}(4 cos(4x) + 9/4 sin(4x)) **A1**

(b) Answer: y = e^{-2x}(4 cos(4x) + 9/4 sin(4x))

Question 7

(a) auxiliary equation (m - 4)^2 = 0 **M1**

(a) repeated root m = 4 **A1**

(a) y = (A + Bx)e^{4x} **A1**

(a) Answer: y = (A + Bx)e^{4x}

(b) y(0) = A = 5 **M1**

(b) y' = (B + 4(A + Bx))e^{4x}, so y'(0) = B + 4A = 4 **M1**

(b) B = -16 **A1**

(b) y = (5 - 16x)e^{4x} **A1**

(b) Answer: y = (5 - 16x)e^{4x}

Question 8

(a) auxiliary equation m^2 + 4m + 3 = 0 **M1**

(a) m = -3, -1 **A1**

(a) y = Ae^{-3x} + Be^{-x} **A1**

(a) Answer: y = Ae^{-3x} + Be^{-x}

(b) substitutes y = ke^{2x}: k(4 + 4 x 2 + 3) = -3 **M1**

(b) 15k = -3 **A1**

(b) k = -1/5 **A1**

(b) Answer: y = (-1/5)e^{2x}

(c) general solution = complementary function + particular integral **B1**

(c) uses y(0) = -2 and y'(0) = 1 **M1**

(c) A = 1/5, B = -2 **A1**

(c) y = (1/5)e^{-3x} + (-2)e^{-x} + (-1/5)e^{2x} **A1**

(c) Answer: y = (1/5)e^{-3x} + (-2)e^{-x} + (-1/5)e^{2x}

Question 9

(a) auxiliary equation $m^2 + 4m + 3 = 0$ **M1**

(a) m = -1, -3 **A1**

(a) $y = Ae^{-x} + Be^{-3x}$ **A1**

(a) Answer: $y = Ae^{-x} + Be^{-3x}$

(b) substitutes $y = ke^{2x}$: $k(4 + 4 \times 2 + 3) = -4$ **M1**

(b) 15k = -4 **A1**

(b) k = -4/15 **A1**

(b) Answer: $y = (-4/15)e^{2x}$

(c) general solution = complementary function + particular integral **B1**

(c) uses y(0) = 3 and y'(0) = -2 **M1**

(c) A = 25/6, B = -9/10 **A1**

(c) $y = (25/6)e^{-x} + (-9/10)e^{-3x} + (-4/15)e^{2x}$ **A1**

(c) Answer: $y = (25/6)e^{-x} + (-9/10)e^{-3x} + (-4/15)e^{2x}$

Question 10

(a) auxiliary equation $m^2-6m+9=0$ written as $(m-3)^2=0$, or discriminant = 36-36 = 0 shown **M1**

(a) repeated root m = 3, cso **A1**

(a) Answer: Repeated root m = 3

(b) $y = (A + Bx)e^{3x}$, ft from part (a) **B1**

(b) Answer: $y = (A + Bx)e^{3x}$

(c) differentiates using the product rule **M1**

(c) correct $dy/dx = (3A+B+3Bx)e^{3x}$ **A1**

(c) substitutes x=0 into y and dy/dx to form equations A=2 and 3A+B=1 **M1**

(c) A = 2, B = -5 (both required) **A1**

(c) $y = (2-5x)e^{3x}$, cao **A1**

(c) Answer: $y = (2 - 5x)e^{3x}$

Question 11

(a) writes auxiliary equation $m^2+4m+13=0$ **M1**

(a) uses the quadratic formula or completes the square: $m = (-4 \pm \sqrt{16-52})/2$ **M1**

(a) m = -2 ± 3i, cso (both roots required, answer printed) **A1**

(a) Answer: m = -2 + 3i or m = -2 - 3i

(b) recognises complex roots p ± qi give solutions of the form $e^{px}(A \cos(qx) + B \sin(qx))$ **M1**

(b) $y = e^{-2x}(A \cos 3x + B \sin 3x)$, cao **A1**

(b) Answer: $y = e^{-2x}(A \cos 3x + B \sin 3x)$

(c) differentiates the general solution using the product rule **M1**

(c) substitutes x=0 into y and dy/dx to form equations A=0 and -2A+3B=6 **M1**

(c) A = 0, B = 2 (both required) **A1**

(c) $y = 2e^{-2x} \sin 3x$, cao **A1**

(c) Answer: $y = 2e^{-2x} \sin 3x$

(d) y oscillates with decreasing amplitude, tending to 0 as x tends to infinity **B1**

(d) correct reason: e^{-2x} tends to 0 while sin3x remains bounded between -1 and 1 **B1**

(d) Answer: y tends to 0 as x tends to infinity (a damped oscillation)

Question 12

(a) differentiates y using the product rule **M1**

(a) correct $dy/dx = -5e^{-x}(\cos 2x + \sin 2x)$ **A1**

(a) differentiates again using the product rule **M1**

(a) correct $d^2y/dx^2 = e^{-x}(-5\cos 2x + 15\sin 2x)$ **A1**

(a) substitutes into $d^2y/dx^2 + 2(dy/dx) + 5y$ and shows the cos2x and sin2x coefficients both sum to 0, cso **A1**

(a) Answer: $d^2y/dx^2 + 2(dy/dx) + 5y = 0$, as required

(b) solves auxiliary equation $m^2+2m+5=0$ using the quadratic formula **M1**

(b) m = -1 ± 2i **A1**

(b) general solution $y = e^{-x}(C \cos 2x + D \sin 2x)$, oe **B1**

(b) Answer: $y = e^{-x}(C \cos 2x + D \sin 2x)$

(c) sets $3\cos 2x - \sin 2x = 0$ and rearranges to $\tan 2x = 3$ **M1**

(c) finds two values of 2x in [0, 2pi] satisfying $\tan 2x = 3$ **M1**

(c) x = 0.625 (3sf) **A1**

(c) x = 2.20 (3sf) **A1**

(c) Answer: x = 0.625 or x = 2.20 (3sf)

Question 13

(a) auxiliary equation $m^2-3m-4=0$ **M1**

(a) factorises to (m-4)(m+1)=0, giving m=4, m=-1 **A1**

(a) complementary function $y = Ae^{4x} + Be^{-x}$, ft **A1**

(a) Answer: CF: $y = Ae^{4x} + Be^{-x}$

(b) differentiates trial twice: $dy/dx=2ax+b$, $d^2y/dx^2=2a$ **M1**

(b) substitutes into the differential equation and compares coefficients of x^2, x and constants **M1**

(b) a = -2 **A1**

(b) b = 3 **A1**

(b) c = -11/4 **A1**

(b) Answer: PI: $y = -2x^2 + 3x - 11/4$

(c) $y = Ae^{4x} + Be^{-x} - 2x^2 + 3x - 11/4$, ft **B1**

(c) Answer: $y = Ae^{4x} + Be^{-x} - 2x^2 + 3x - 11/4$

(d) substitutes x=0, y=0 into the general solution to form A+B-11/4=0 **M1**

(d) differentiates and substitutes x=0, dy/dx=0 to form 4A-B+3=0 **M1**

(d) A = -1/20 (or B = 14/5) **A1**

(d) A = -1/20 and B = 14/5, both stated in $y = -1/20\ e^{4x} + 14/5\ e^{-x} - 2x^2 + 3x - 11/4$, cao **A1**

(d) Answer: $y = -1/20\ e^{4x} + 14/5\ e^{-x} - 2x^2 + 3x - 11/4$

Question 14

(a) auxiliary equation $m^2-4m+3=0$ **M1**

(a) factorises to (m-1)(m-3)=0, giving m=1, m=3, cso **A1**

(a) complementary function $y = Ae^x + Be^{3x}$ **B1**

(a) Answer: CF: $y = Ae^x + Be^{3x}$

(b) e^{3x} is already part of the complementary function (a solution of the homogeneous equation), so the standard trial would give zero and cannot match the right hand side **B1**

(b) states the correct alternative trial $y = Cxe^{3x}$ **B1**

(b) Answer: Try $y = Cxe^{3x}$

(c) differentiates $y=Cxe^{3x}$ using the product rule to find dy/dx **M1**

(c) correct $dy/dx = Ce^{3x}(1+3x)$ and $d^2y/dx^2 = Ce^{3x}(6+9x)$ **A1**

(c) substitutes into the differential equation and simplifies **M1**

(c) obtains $2Ce^{3x} = e^{3x}$ **A1**

(c) C = 1/2, cao **A1**

(c) Answer: PI: $y = (1/2)xe^{3x}$

(d) $y = Ae^x + Be^{3x} + (1/2)xe^{3x}$, ft **B1**

(d) Answer: $y = Ae^x + Be^{3x} + (1/2)xe^{3x}$

Question 15

(a) auxiliary equation $m^2+1=0$ gives m = +-i **B1**

(a) complementary function y = A cosx + B sinx **B1**

(a) Answer: CF: y = A cosx + B sinx

(b) differentiates trial twice: $d^2y/dx^2 = -4Pcos2x - 4Qsin2x$ **M1**

(b) substitutes into d^2y/dx^2+y and compares coefficients: $-3Pcos2x-3Qsin2x = sin2x$ **M1**

(b) P = 0 **A1**

(b) Q = -1/3 **A1**

(b) particular integral y = -(1/3)sin2x correctly stated **A1**

(b) Answer: P = 0, Q = -1/3

(c) y = A cosx + B sinx - (1/3)sin2x, ft **B1**

(c) Answer: y = A cosx + B sinx - (1/3)sin2x

(d) differentiates the general solution: dy/dx = -Asinx+Bcosx-(2/3)cos2x **M1**

(d) correct derivative **A1**

(d) substitutes x=0, y=1 to obtain A = 1 **M1**

(d) substitutes x=0, dy/dx=0 to obtain B = 2/3 **M1**

(d) y = cosx + (2/3)sinx - (1/3)sin2x, cao **A1**

(d) Answer: y = cosx + (2/3)sinx - (1/3)sin2x

Question 16

(a) substitutes the given expressions for $x^2(d^2y/dx^2)$ and x(dy/dx) into the differential equation **M1**

(a) obtains $(d^2y/du^2 - dy/du) + (dy/du) - 4y = 0$ with the -dy/du and +dy/du terms identified **A1**

(a) correctly cancels to give $d^2y/du^2 - 4y = 0$, cso (answer printed) **A1**

(a) Answer: $d^2y/du^2 - 4y = 0$

(b) auxiliary equation $m^2-4=0$ solved to give m = ±2 **M1**

(b) $y = Ae^{2u} + Be^{-2u}$ **A1**

(b) uses u = ln x to write $e^{2u} = x^2$ and $e^{-2u} = x^{-2}$ **M1**

(b) $y = Ax^2 + B/x^2$, cso **A1**

(b) Answer: $y = Ax^2 + B/x^2$

(c) differentiates $y=Ax^2+B/x^2$ to obtain $dy/dx=2Ax-2B/x^3$ **M1**

(c) substitutes x=1 into y and dy/dx to form A+B=3 and 2A-2B=-1 **M1**

(c) A = 5/4 (or 1.25) **A1**

(c) B = 7/4 (or 1.75) **A1**

(c) $y = (5/4)x^2 + (7/4)/x^2$, cao **A1**

(c) Answer: $y = (5/4)x^2 + (7/4)/x^2$

Question 17

(a) rearranges dx/dt = x+2y to express y in terms of x and dx/dt: y = (dx/dt - x)/2 **M1**

(a) differentiates dx/dt = x+2y and substitutes dy/dt = 3x+2y to eliminate dy/dt **M1**

(a) substitutes the expression for y (or 4y) to eliminate the remaining y term **A1**

(a) obtains $d^2x/dt^2 - 3(dx/dt) - 4x = 0$, cso (answer printed) **A1**

(a) Answer: $d^2x/dt^2 - 3(dx/dt) - 4x = 0$

(b) auxiliary equation $m^2-3m-4=0$ solved to give m=4, m=-1 **M1**

(b) $x = Ae^{4t} + Be^{-t}$ **A1**

(b) uses x(0)=5 and y(0)=0 (via y=(dx/dt-x)/2 from part a) to form simultaneous equations in A and B **M1**

(b) A = 2, B = 3 (both required) **A1**

(b) $x = 2e^{4t} + 3e^{-t}$, cao **A1**

(b) Answer: $x = 2e^{4t} + 3e^{-t}$

(c) differentiates x(t) found in part (b): $dx/dt = 8e^{4t} - 3e^{-t}$ **M1**

(c) substitutes into y = (dx/dt - x)/2 from part (a) **M1**

(c) $y = 3e^{4t} - 3e^{-t}$, cao **A1**

(c) Answer: $y = 3e^{4t} - 3e^{-t}$

Question 18

(a) auxiliary equation $m^2+6m+25=0$ **M1**

(a) uses the quadratic formula: $m = (-6 \pm \sqrt{36-100})/2$ **M1**

(a) $m = -3 \pm 4i$, complementary function $x = e^{-3t}(A\cos 4t+B\sin 4t)$, cso **A1**

(a) Answer: CF: $x = e^{-3t}(A \cos 4t + B \sin 4t)$

(b) differentiates trial twice: $d^2x/dt^2 = -P\cos t - Q\sin t$ **M1**

(b) substitutes into the differential equation and compares coefficients of cos t and sin t **M1**

(b) forms the pair of equations $24P+6Q=51$ and $-6P+24Q=0$, oe **A1**

(b) $P = 2$, cso **A1**

(b) $Q = 0.5$, cso **A1**

(b) Answer: P = 2, Q = 0.5

(c) differentiates the general solution $x=e^{-3t}(A\cos 4t+B\sin 4t)+2\cos t+0.5\sin t$ **M1**

(c) substitutes t=0, x=0 to obtain $A+2=0$, so $A=-2$ **M1**

(c) substitutes t=0, dx/dt=0 to obtain $-3A+4B+0.5=0$ **M1**

(c) $B = -13/8$ (or -1.625) **A1**

(c) $x = e^{-3t}(-2\cos 4t - 1.625\sin 4t) + 2\cos t + 0.5\sin t$, cao **A1**

(c) Answer: $x = e^{-3t}(-2\cos 4t - 1.625\sin 4t) + 2\cos t + 0.5\sin t$

(d) the transient part is $e^{-3t}(-2\cos 4t - 1.625\sin 4t)$, which decays to zero as t becomes large because of the e^{-3t} factor **B1**

(d) the steady state part is $2\cos t + 0.5\sin t$, which persists indefinitely, oscillating with the same period as the forcing term **B1**

(d) as t becomes large, x(t) approaches the steady state part $2\cos t + 0.5\sin t$ since the transient part dies away **B1**

(d) Answer: Transient: $e^{-3t}(-2\cos 4t-1.625\sin 4t)$; Steady state: $2\cos t+0.5\sin t$