1.9 Proof by Induction: 10-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 10 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Let $T(n) = 1^3 + 2^3 + 3^3 + ... + n^3$.

(a) Verify that the statement $T(n) = [n(n+1)/2]^2$ is true for $n = 1$. **(1)**

(b) Prove by induction that $T(n) = [n(n+1)/2]^2$ for all positive integers n. **(4)**

(c) Show that $1^3 + 2^3 + ... + 10^3 = 3025$, and verify that this equals $(1 + 2 + ... + 10)^2$. **(2)**

(Total for Question 1 is 7 marks)

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Mark scheme · 1.9 Proof by Induction: 10-minute Pack

Question 1

(a) LHS = 1, RHS = $[1*2/2]^2$ = 1, so true for n=1 **B1**

(a) Answer: n = 1: LHS = 1, RHS = 1, statement holds.

(b) assume true for n=k: T(k) = $[k(k+1)/2]^2$, form T(k+1) = T(k) + $(k+1)^3$ **M1**

(b) take out common factor $(k+1)^2$: T(k+1) = $(k+1)^2[\ k^2/4 + (k+1)\ ]$ **dM1**

(b) correct simplification to $[(k+1)(k+2)/2]^2$ cso **A1**

(b) conclusion: true for n=1, true for n=k implies true for n=k+1, so true for all positive integers n by mathematical induction **A1**

(b) Answer: T(k+1) = $[(k+1)(k+2)/2]^2$; true for all positive integers n by induction.

(c) $[10(11)/2]^2 = 55^2$ = 3025 cso **B1**

(c) 1+2+...+10 = 55, and 55^2 = 3025, so the two are equal **B1**

(c) Answer: 3025; equals $(55)^2$, confirming the identity.

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