1.9 Proof by Induction: 30-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Let U(n) = 1(2) + 2(3) + 3(4) + ... + n(n+1).

(a) Verify that the statement U(n) = (1/3)n(n+1)(n+2) is true for n = 1. **(1)**

(b) Prove by induction that U(n) = (1/3)n(n+1)(n+2) for all positive integers n. **(4)**

(c) Hence find the value of 1(2) + 2(3) + ... + 15(16). **(2)**

(Total for Question 1 is 7 marks)

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2 Consider the statement P(n): 3^{2n} - 1 is divisible by 8, for all positive integers n.

(a) Verify that P(1) is true. **(1)**

(b) Prove by induction that 3^{2n} - 1 is divisible by 8 for all positive integers n. **(5)**

(c) By testing the case n = 2, determine whether the statement '3^{2n} - 1 is divisible by 24 for all positive integers n' is true or false. **(2)**

(Total for Question 2 is 8 marks)

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3 The sequence u_n is defined by $u_1 = 2$ and $u_{n+1} = 3u_n + 2$. Prove by induction that $u_n = 3 \times 3^{n-1} - 1$ for all positive integers n.

(a) Show that the formula gives the correct value for n = 1. **(1)**

(b) Assume true for n = k and prove it is true for n = k + 1, and state the conclusion. **(4)**

(Total for Question 3 is 5 marks)

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4 Prove by mathematical induction that, for all positive integers n, the sum from r = 1 to n of r(r + 5) is equal to n(n + 1)(2n + 16)/6.

(a) State the base case n = 1 and verify the result is true for n = 1. **(1)**

(b) Assume the result is true for n = k, and prove it is then true for n = k + 1. Hence state the conclusion. **(4)**

(Total for Question 4 is 5 marks)

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Mark scheme · 1.9 Proof by Induction: 30-minute Pack

Question 1

(a) LHS = 1(2) = 2, RHS = (1/3)(1)(2)(3) = 2, so true for n=1 **B1**

(a) Answer: n = 1: LHS = 2, RHS = 2, statement holds.

(b) assume true for n=k: U(k) = (1/3)k(k+1)(k+2), form U(k+1) = U(k) + (k+1)(k+2) **M1**

(b) take out common factor (k+1)(k+2): U(k+1) = (k+1)(k+2)[k/3 + 1] **dM1**

(b) correct simplification to (1/3)(k+1)(k+2)(k+3) cso **A1**

(b) conclusion: true for n=1, true for n=k implies true for n=k+1, so true for all positive integers n by mathematical induction **A1**

(b) Answer: U(k+1) = (1/3)(k+1)(k+2)(k+3); true for all positive integers n by induction.

(c) substitute n=15 into (1/3)n(n+1)(n+2) **M1**

(c) 1360 cao **A1**

(c) Answer: 1360

Question 2

(a) $3^2 - 1 = 8 = 8(1)$, which is divisible by 8 **B1**

(a) Answer: n = 1: $3^2 - 1 = 8$, divisible by 8.

(b) assume true for n=k: $3^{2k} - 1 = 8m$ for some positive integer m, so $3^{2k} = 8m + 1$ **M1**

(b) write $3^{2(k+1)} - 1 = 9(3^{2k}) - 1$ **M1**

(b) substitute $3^{2k} = 8m+1$ to get 9(8m+1) - 1 **dM1**

(b) simplify to 72m + 8 = 8(9m+1), which is divisible by 8, cso **A1**

(b) conclusion: true for n=1, true for n=k implies true for n=k+1, so true for all positive integers n by mathematical induction **A1**

(b) Answer: $3^{2(k+1)} - 1 = 8(9m+1)$; true for all positive integers n by induction.

(c) n=2: $3^4 - 1 = 80$ **B1**

(c) 80 / 24 is not an integer (80 = 24(3) + 8), so the statement is false **B1**

(c) Answer: False: $3^4 - 1 = 80$, which is not divisible by 24.

Question 3

(a) $u_1 = 3 \times 3^0 - 1 = 2$ **B1**

(a) Answer: true for n = 1

(b) $u_{k+1} = 3u_k + 2 = 3(3 \times 3^{k-1} - 1) + 2$ **M1**

(b) expands: $3 \times 3^k - 3 + 2$ **M1**

(b) 2 - 3 = -1, giving $3 \times 3^k - 1$ **A1**

(b) true for n = 1, true for n = k implies true for n = k + 1, so true for all n by induction (cso) **A1**

(b) Answer: proved by induction

Question 4

(a) LHS = 6, RHS = 6, so true for n = 1 **B1**

(a) Answer: n = 1: both sides equal 6

(b) assumes sum to k = k(k + 1)(2k + 16)/6 and adds the (k + 1)th term **M1**

(b) factorises or expands to simplify **M1**

(b) obtains (k + 1)((k + 1) + 1)(2(k + 1) + 16)/6 by the simplification: k(k + 1)(2k + 16)/6 + (k + 1)(k + 6) = (k + 1)[k(2k + 16) + 6(k + 6)]/6 = (k + 1)(k + 2)(2k + 18)/6 **A1**

(b) true for n = 1, and true for n = k implies true for n = k + 1, so true for all positive integers n by induction (cso) **A1**

(b) Answer: result proved by induction

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