4.1 Decision Maths: Algorithms, Sorting and Bin Packing: 10-minute Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 10 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question uses known comparison-count formulas to compare the efficiency of a bubble sort and a quicksort as the size of the list, n, grows large.

(a) The number of comparisons made by a bubble sort in the worst case, on a list of n items, is given by n(n - 1)/2. Calculate the worst-case number of comparisons for (i) n = 50, (ii) n = 200, (iii) n = 1024. **(3)**

(b) Given that n increased by a factor of 4 between parts (a)(i) and (a)(ii), show that the ratio of your two answers is consistent with the bubble sort's worst-case order of $O(n^2)$. **(2)**

(c) A quicksort, on average, makes approximately n log2(n) comparisons to sort a list of n items. Calculate the approximate number of comparisons a quicksort would make, on average, to sort a list of n = 1024 items. **(2)**

(d) By comparing your answer to part (c) with your answer to part (a)(iii), comment on the practical significance, for large n, of the difference in order between an O(n log n) algorithm and an $O(n^2)$ algorithm. **(2)**

(Total for Question 1 is 9 marks)

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Mark scheme · 4.1 Decision Maths: Algorithms, Sorting and Bin Packing: 10-minute Pack

Question 1

(a) (i) 1225 cao (50 x 49 / 2) **B1**

(a) (ii) 19900 cao (200 x 199 / 2) **B1**

(a) (iii) 523776 cao (1024 x 1023 / 2) **B1**

(a) Answer: (i) 1225; (ii) 19900; (iii) 523776.

(b) correctly calculates the ratio 19900 / 1225 = 16.24... (awrt 16.2) **M1**

(b) compares this to $4^2 = 16$ and concludes the closeness confirms the order $O(n^2)$ (the small difference from exactly 16 is because n(n-1)/2 also has a lower-order term, becoming negligible for large n) (oe) **A1**

(b) Answer: Ratio approx 16.2, close to $4^2 = 16$, consistent with worst-case order $O(n^2)$.

(c) uses log2(1024) = 10 (since $2^{10} = 1024$) **M1**

(c) 1024 x 10 = 10240 comparisons cao **A1**

(c) Answer: Approximately 10240 comparisons.

(d) for n = 1024, quicksort's average 10240 comparisons is far smaller than bubble sort's worst-case 523776 comparisons, roughly 51 times fewer (oe, allow awrt 50) **B1**

(d) as n grows large, an O(n log n) algorithm remains practical while an $O(n^2)$ algorithm's run time grows far more quickly, so the difference in order becomes increasingly significant for large data sets (oe) **B1**

(d) Answer: Quicksort's estimate (10240) is roughly 51 times smaller than bubble sort's worst case (523776); as n grows, this gap widens further, so the choice of algorithm order matters far more for large data sets.

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